

CHSE Odisha · +2 Science

MATHEMATICS

+2 1st Year (Class XI) · As per CHSE Syllabus 2023-24

Theory in simple language · Formula boxes
Solved problems · Exam Q&A · Practice & revision

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MATHEMATICS

+2 1st Year (Class XI)

Exam Pattern & How to Study

Mathematics - Unit Weightage

Class XI Unit	Marks	Class XII Unit	Marks
Sets and Functions	23	Relations and Functions	08
Algebra	25	Algebra (Matrices, Determinants)	10
Coordinate Geometry	12	Calculus	35
Calculus	08	Vectors and 3-D Geometry	14
Statistics and Probability	12	Linear Programming	05
		Probability	08

Question typology: about 55% Remembering/Understanding, 25% Applying, 20% Analysing/Evaluating/Creating. There is no overall choice, but about 33% internal choice in every section.

GOLDEN RULES FOR WRITING ANSWERS

- Always write the **formula first**, then substitute values, then the answer **with units**.
- Underline or box the final answer. Examiners mark step-by-step, so even a wrong final answer earns step marks.
- For theory answers, use short points and draw a diagram wherever possible.
- Attempt questions you know best first, but keep their numbering exactly as in the paper.

XI-1. Sets

Idea in simple words: A set is just a well-defined collection of objects. "Well-defined" means everybody agrees whether an object is in the collection or not. "Vowels of English" is a set; "beautiful flowers" is not (people disagree on what is beautiful).

KEY DEFINITIONS

- **Element:** an object in a set. If 3 is in A we write $3 \in A$; if 7 is not, $7 \notin A$.
- **Roster (list) form:** $A = \{a, e, i, o, u\}$. **Set-builder form:** $A = \{x : x \text{ is a vowel}\}$.
- **Empty (null) set ϕ or $\{\}$:** has no element, e.g. $\{x : x \text{ is a natural number less than } 1\}$.
- **Finite set:** elements can be counted. **Infinite set:** elements never end, e.g. $N = \{1, 2, 3, \dots\}$.
- **Equal sets:** exactly the same elements. $\{1, 2, 3\} = \{3, 1, 2\}$ (order and repetition don't matter).
- **Subset:** $A \subset B$ if every element of A is also in B. Every set is a subset of itself; ϕ is a subset of every set.
- **Power set $P(A)$:** the set of all subsets of A.
- **Universal set U:** the "big" set containing all sets under discussion.

Intervals (subsets of real numbers)

Notation	Meaning	Name
(a, b)	$\{x : a < x < b\}$	Open interval (ends not included)
$[a, b]$	$\{x : a \leq x \leq b\}$	Closed interval (ends included)
$[a, b)$	$\{x : a \leq x < b\}$	Half-open
$(a, b]$	$\{x : a < x \leq b\}$	Half-closed

Operations on sets

FORMULAS TO REMEMBER

$$A \cup B = \{x : x \in A \text{ or } x \in B\} \quad A \cap B = \{x : x \in A \text{ and } x \in B\}$$

$$A - B = \{x : x \in A, x \notin B\} \quad A' = U - A \text{ (complement)}$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$\text{If } n(A) = m, \text{ number of subsets} = 2^m; \text{ number of proper subsets} = 2^m - 1$$

$$\text{De Morgan's laws: } (A \cup B)' = A' \cap B' \text{ and } (A \cap B)' = A' \cup B'$$

$$\text{Complement laws: } A \cup A' = U, A \cap A' = \phi, (A')' = A, U' = \phi, \phi' = U$$

Venn diagram: U is drawn as a rectangle and sets as circles inside it. The overlapping part of two circles is $A \cap B$. Shading regions is the easiest way to check any set identity.

Example 1. Write $A = \{x : x \text{ is an integer, } -3 \leq x < 2\}$ in roster form.

Integers from -3 up to (but not including) 2 : **$A = \{-3, -2, -1, 0, 1\}$**

Example 2. If $A = \{1, 2\}$, write $P(A)$.

Subsets: $\phi, \{1\}, \{2\}, \{1, 2\}$. So $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$, which has $2^2 = 4$ elements.

Example 3. $U = \{1, 2, \dots, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$. Find $A \cup B$, $A \cap B$, $A - B$, $(A \cup B)'$.

$A \cup B = \{1, 2, 3, 4, 6, 8\}$; $A \cap B = \{2, 4\}$; $A - B = \{1, 3\}$; $(A \cup B)' = U - (A \cup B) = \{5, 7, 9\}$.

Check De Morgan: $A' = \{5, 6, 7, 8, 9\}$, $B' = \{1, 3, 5, 7, 9\}$, $A' \cap B' = \{5, 7, 9\}$ ✓.

Example 4. In a class of 60 students, 35 like tea, 25 like coffee and 10 like both. How many like neither?

$n(T \cup C) = 35 + 25 - 10 = 50$. Neither = $60 - 50 = 10$ students.

Example 5. In a survey of 400 people, 200 read paper H, 150 read paper E, 100 read paper O; 50 read H and E, 30 read E and O, 40 read H and O, and 20 read all three. How many read at least one paper? How many read only H?

$n(H \cup E \cup O) = 200 + 150 + 100 - 50 - 30 - 40 + 20 = 350$.

Only H = $n(H) - n(H \cap E) - n(H \cap O) + n(H \cap E \cap O) = 200 - 50 - 40 + 20 = 130$.

XI-2. Relations and Functions

Idea: A relation tells us which elements of one set are "connected" to elements of another set. A function is a special relation in which every input has exactly one output — like a vending machine: press one button, get one item.

DEFINITIONS

- **Ordered pair (a, b) :** order matters, $(2, 3) \neq (3, 2)$. $(a, b) = (c, d) \Leftrightarrow a = c$ and $b = d$.
- **Cartesian product:** $A \times B = \{(a, b) : a \in A, b \in B\}$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.
- $R \times R = \{(x, y) : x, y \text{ real}\}$ is the coordinate plane; $R \times R \times R$ is 3-D space.
- **Relation** from A to B: any subset of $A \times B$. Number of relations = 2^{pq} .
- **Domain:** set of first elements; **Range:** set of second elements actually used; **Codomain:** the whole set B.
- **Function $f : A \rightarrow B$:** a relation in which every element of A has one and only one image in B.

Important real functions and their graphs

Function	Rule	Domain	Range	Graph shape
Identity	$f(x) = x$	$\mathbb{R}$	$\mathbb{R}$	Straight line through origin at 45°
Constant	$f(x) = c$	$\mathbb{R}$	$\{c\}$	Horizontal line
Polynomial	$a_0 + a_1x + \dots + a_nx^n$	$\mathbb{R}$	depends	Smooth curve (x^2 is a U-shaped parabola)
Rational	$p(x)/q(x)$	$\mathbb{R} - \{x : q(x) = 0\}$	depends	e.g. $1/x$ is a hyperbola
Modulus	$ x = x$ if $x \geq 0$; $-x$ if $x < 0$	$\mathbb{R}$	$[0, \infty)$	V-shape
Signum	1 if $x > 0$; 0 if $x = 0$; -1 if $x < 0$	$\mathbb{R}$	$\{-1, 0, 1\}$	Two horizontal rays and a dot
Greatest integer	$[x] = \text{greatest integer} \leq x$	$\mathbb{R}$	$\mathbb{Z}$	Staircase
Exponential	a^x ($a > 0, a \neq 1$)	$\mathbb{R}$	$(0, \infty)$	Always above x-axis, passes $(0,1)$
Logarithmic	$\log_a x$	$(0, \infty)$	$\mathbb{R}$	Passes $(1, 0)$

QUICK EXAMPLES OF $[x]$

$[2.7] = 2$, $[-2.7] = -3$, $[5] = 5$, $[-0.1] = -1$.

Algebra of functions: $(f \pm g)(x) = f(x) \pm g(x)$; $(fg)(x) = f(x) \cdot g(x)$; $(f/g)(x) = f(x)/g(x)$, where $g(x) \neq 0$.

Example 1. If $(x + 1, y - 2) = (3, 1)$, find x and y .

$x + 1 = 3 \Rightarrow x = 2$; $y - 2 = 1 \Rightarrow y = 3$. **$x = 2, y = 3$**

Example 2. $A = \{1, 2\}$, $B = \{3, 4, 5\}$. Find $A \times B$ and the number of relations from A to B .

$A \times B = \{(1,3), (1,4), (1,5), (2,3), (2,4), (2,5)\}$, $n = 6$. Number of relations = $2^6 = \mathbf{64}$.

Example 3. $R = \{(x, x + 5) : x \in \{0,1,2,3,4,5\}\}$. Write R in roster form, its domain and range.

$R = \{(0,5), (1,6), (2,7), (3,8), (4,9), (5,10)\}$. Domain = $\{0,1,2,3,4,5\}$; Range = **$\{5,6,7,8,9,10\}$** .

Example 4. Find the domain of $f(x) = (x^2 + 2x + 1)/(x^2 - 8x + 12)$.

Denominator = 0 when $x^2 - 8x + 12 = 0 \Rightarrow (x - 2)(x - 6) = 0 \Rightarrow x = 2, 6$. Domain = **$\mathbb{R} - \{2, 6\}$** .

Example 5. Find the domain and range of $f(x) = \sqrt{x - 1}$.

Square root needs $x - 1 \geq 0 \Rightarrow x \geq 1$. Domain = $[1, \infty)$. A square root is never negative, so Range = $[0, \infty)$.

Example 6. $f(x) = x^2$, $g(x) = 2x + 1$. Find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, $(f/g)(x)$.

$(f + g)(x) = x^2 + 2x + 1$; $(f - g)(x) = x^2 - 2x - 1$; $(fg)(x) = 2x^3 + x^2$; $(f/g)(x) = x^2/(2x + 1)$, $x \neq -1/2$.

COMMON MISTAKE

A relation in which one input has two outputs, e.g. $\{(1,2),(1,3)\}$, is **not** a function. But two inputs with the same output, e.g. $\{(1,5),(2,5)\}$, **is** a function.

XI-3. Trigonometric Functions

Idea: Trigonometry studies angles and the ratios of sides of triangles. It is used to find heights of towers, distances of ships, and to describe waves and sound.

Measuring angles

DEGREE-RADIAN CONVERSION

π radians = $180^\circ \Rightarrow 1 \text{ radian} \approx 57^\circ 16'$ $1^\circ = \pi/180 \text{ radian} \approx 0.01746 \text{ rad}$

Radian = Degree $\times \pi/180$ Degree = Radian $\times 180/\pi$

Arc length: $l = r\theta$ (θ in radians)

An angle measured anticlockwise is **positive**; clockwise is **negative**.

Definition with the unit circle

Draw a circle of radius 1 with centre at the origin. If a point P on the circle makes angle x with the positive x -axis, then $P = (\cos x, \sin x)$. Since P is on the circle, $(\cos x)^2 + (\sin x)^2 = 1$. This proves **$\sin^2 x + \cos^2 x = 1$ for all x** .

Angle	0°	30°	45°	60°	90°	180°	270°	360°
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	-1	0
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	-1	0	1
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined	0	not defined	0

SIGNS IN FOUR QUADRANTS - "ALL SILVER TEA CUPS"

Quadrant I: All positive · II: Sin (and cosec) positive · III: Tan (and cot) positive · IV: Cos (and sec) positive.

Function	Domain	Range
$\sin x$	$\mathbb{R}$	$[-1, 1]$
$\cos x$	$\mathbb{R}$	$[-1, 1]$
$\tan x$	$\mathbb{R} - \{(2n+1)\pi/2\}$	$\mathbb{R}$
$\cot x$	$\mathbb{R} - \{n\pi\}$	$\mathbb{R}$
$\sec x$	$\mathbb{R} - \{(2n+1)\pi/2\}$	$\mathbb{R} - (-1, 1)$
$\operatorname{cosec} x$	$\mathbb{R} - \{n\pi\}$	$\mathbb{R} - (-1, 1)$

COMPOUND-ANGLE AND OTHER IDENTITIES (MUST MEMORISE)

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = (\tan x \pm \tan y) / (1 \mp \tan x \tan y)$$

$$\cot(x \pm y) = (\cot x \cot y \mp 1) / (\cot y \pm \cot x)$$

$$\sin \alpha + \sin \beta = 2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta) \quad \sin \alpha - \sin \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta) \quad \cos \alpha - \cos \beta = -2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)$$

$$\sin 2x = 2 \sin x \cos x = 2 \tan x / (1 + \tan^2 x)$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x = (1 - \tan^2 x) / (1 + \tan^2 x)$$

$$\tan 2x = 2 \tan x / (1 - \tan^2 x)$$

$$\sin 3x = 3 \sin x - 4 \sin^3 x \quad \cos 3x = 4 \cos^3 x - 3 \cos x \quad \tan 3x = (3 \tan x - \tan^3 x) / (1 - 3 \tan^2 x)$$

$$1 + \tan^2 x = \sec^2 x \quad 1 + \cot^2 x = \operatorname{cosec}^2 x$$

Example 1. Convert $40^\circ 20'$ into radians.

$$40^\circ 20' = 40\frac{1}{3}^\circ = 121/3 \text{ degrees. Radians} = (121/3) \times \pi/180 = \mathbf{121\pi/540}.$$

Example 2. Convert 6 radians into degrees (use $\pi = 22/7$).

$$6 \times 180/\pi = 6 \times 180 \times 7/22 = 7560/22 = 343.636^\circ \approx \mathbf{343^\circ 38' 11''}.$$

Example 3. A wheel makes 360 revolutions per minute. Through how many radians does it turn in one second?

$$360 \text{ rev/min} = 6 \text{ rev/second. } 1 \text{ revolution} = 2\pi \text{ rad} \Rightarrow 6 \times 2\pi = \mathbf{12\pi \text{ radians}}.$$

Example 4. If $\cos x = -3/5$ and x lies in the third quadrant, find the other five ratios.

$$\sin^2 x = 1 - 9/25 = 16/25 \Rightarrow \sin x = -4/5 \text{ (negative in Q III). } \tan x = \sin/\cos = 4/3, \cot x = 3/4, \sec x = -5/3, \operatorname{cosec} x = -5/4.$$

Example 5. Find $\sin 75^\circ$ and $\cos 15^\circ$.

$$\sin 75^\circ = \sin(45^\circ + 30^\circ) = \sin 45 \cos 30 + \cos 45 \sin 30 = (1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(1/2) = \mathbf{(\sqrt{3} + 1)/(2\sqrt{2})}.$$

Since $\cos 15^\circ = \sin 75^\circ$, $\cos 15^\circ$ has the same value.

Example 6. Prove: $(\sin 5x + \sin 3x)/(\cos 5x + \cos 3x) = \tan 4x$.

$$\text{Numerator} = 2 \sin 4x \cos x; \text{Denominator} = 2 \cos 4x \cos x. \text{ Dividing: } \sin 4x/\cos 4x = \tan 4x. \text{ Hence proved.}$$

Example 7. Prove: $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$.

$\cos 4x = 1 - 2 \sin^2 2x = 1 - 2(2 \sin x \cos x)^2 = 1 - 8 \sin^2 x \cos^2 x$. Hence proved.

Example 8. Find the value of $\tan 15^\circ$.

$\tan(45^\circ - 30^\circ) = (1 - 1/\sqrt{3})/(1 + 1/\sqrt{3}) = (\sqrt{3} - 1)/(\sqrt{3} + 1)$. Multiply top and bottom by $(\sqrt{3} - 1)$: $(4 - 2\sqrt{3})/2 = 2 - \sqrt{3}$.

Example 9. Find the value of $\sin(31\pi/3)$.

$31\pi/3 = 10\pi + \pi/3$. Since $\sin$ repeats every 2π , $\sin(31\pi/3) = \sin(\pi/3) = \sqrt{3}/2$.

Exam Q&A - Sets, Functions, Trigonometry

Q1. Is the collection of "five most talented writers of India" a set? [1]

No. "Talented" is not well-defined; different people will choose differently.

Q2. How many elements does $P(P(\varphi))$ have? [1]

$P(\varphi) = \{\varphi\}$ has 1 element, so $P(P(\varphi))$ has $2^1 = 2$ elements: $\{\varphi, \{\varphi\}\}$.

Q3. Find the range of $f(x) = |x - 1|$. [2]

A modulus is never negative and takes value 0 at $x = 1$, so range = $[0, \infty)$.

Q4. Find the value of $[-3.5] + [3.5]$. [1]

$[-3.5] = -4$ and $[3.5] = 3$, so the sum is -1 .

Q5. Find the radius of a circle in which a central angle of 60° cuts off an arc of length 37.4 cm ($\pi = 22/7$). [2]

$\theta = 60^\circ = \pi/3$. $r = l/\theta = 37.4 \times 3/\pi = 37.4 \times 3 \times 7/22 = 35.7$ cm.

Q6. Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$. [4]

LHS = $(\sin 6x + \sin 4x)(\sin 6x - \sin 4x) = (2 \sin 5x \cos x)(2 \cos 5x \sin x) = (2 \sin 5x \cos 5x)(2 \sin x \cos x) = \sin 10x \cdot \sin 2x = \text{RHS}$.

Q7. Prove that $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$. [4]

$\cos 6x = \cos 3(2x) = 4\cos^3 2x - 3\cos 2x$. Put $\cos 2x = 2\cos^2 x - 1 = c$: $4c^3 - 3c$. Expanding $4(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1) = 4(8\cos^6 x - 12\cos^4 x + 6\cos^2 x - 1) - 6\cos^2 x + 3 = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$.

Q8. Show that $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$. [4]

$\tan 3x = \tan(2x + x) = (\tan 2x + \tan x)/(1 - \tan 2x \tan x)$. Cross-multiply: $\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$. Rearranging gives the result.

XI-4. Complex Numbers and Quadratic Equations

Idea: The equation $x^2 + 1 = 0$ has no real solution because no real number squared gives -1 . Mathematicians invented the number $i = \sqrt{-1}$ so that every quadratic equation has a solution. Numbers of the form $a + ib$ are called complex numbers.

DEFINITIONS

- **Complex number:** $z = a + ib$, where a, b are real. $a = \text{Re}(z)$ (real part), $b = \text{Im}(z)$ (imaginary part).
- Powers of i : $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, and then the cycle repeats. So i^n depends on the remainder when n is divided by 4.
- **Conjugate:** $\bar{z} = a - ib$. **Modulus:** $|z| = \sqrt{a^2 + b^2}$.
- **Argand plane:** plot $z = a + ib$ as the point (a, b) . The x-axis is the real axis, the y-axis is the imaginary axis. $|z|$ is the distance of the point from the origin.

ALGEBRA OF COMPLEX NUMBERS

$$(a + ib) + (c + id) = (a + c) + i(b + d)$$

$$(a + ib)(c + id) = (ac - bd) + i(ad + bc)$$

$$1/z = \bar{z}/|z|^2 \quad z \cdot \bar{z} = |z|^2$$

$$|z_1 z_2| = |z_1| |z_2| \quad \text{conj}(z_1 + z_2) = \bar{z}_1 + \bar{z}_2$$

Roots of $ax^2 + bx + c = 0$: $x = [-b \pm \sqrt{b^2 - 4ac}]/2a$. If $D = b^2 - 4ac < 0$, roots are $[-b \pm i\sqrt{4ac - b^2}]/2a$

Example 1. Find i^{-39} .

$i^{-39} = 1/i^{39}$. $39 = 4 \times 9 + 3$ so $i^{39} = i^3 = -i$. Then $1/(-i) = -1/i = -i/i^2 = i$.

Example 2. Express $(5 - 3i)^3$ in the form $a + ib$.

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$: $125 - 3(25)(3i) + 3(5)(9i^2) - 27i^3 = 125 - 225i - 135 + 27i = -10 - 198i$.

Example 3. Find the multiplicative inverse of $4 - 3i$.

$$1/z = \bar{z}/|z|^2 = (4 + 3i)/(16 + 9) = 4/25 + (3/25)i.$$

Example 4. Express $(3 + 2i)/(1 - i)$ in $a + ib$ form.

Multiply by $(1 + i)/(1 + i)$: $(3 + 3i + 2i + 2i^2)/(1 + 1) = (1 + 5i)/2 = 1/2 + (5/2)i$.

Example 5. Solve $x^2 + x + 1 = 0$.

$D = 1 - 4 = -3$. $x = (-1 \pm i\sqrt{3})/2 = -1/2 \pm (\sqrt{3}/2)i$.

Example 6. Solve $\sqrt{2}x^2 + x + \sqrt{2} = 0$.

$$D = 1 - 4(\sqrt{2})(\sqrt{2}) = 1 - 8 = -7. \quad x = (-1 \pm i\sqrt{7})/(2\sqrt{2}).$$

XI-5. Linear Inequalities

Idea: An inequality compares two expressions using $<$, $>$, $\leq$ or $\geq$. "You must be at least 18 to vote" is the inequality $\text{age} \geq 18$. We solve inequalities almost like equations, with **one special rule**.

THE ONE SPECIAL RULE

When you multiply or divide both sides by a **negative** number, the inequality sign **reverses**. Example: $-2x > 6 \Rightarrow x < -3$. Adding/subtracting any number, or multiplying by a positive number, keeps the sign the same.

Number-line representation: use an open circle $\circ$ for $<$ or $>$ (point not included), and a filled circle $\bullet$ for $\leq$ or $\geq$ (point included), then shade the correct side.

Example 1. Solve $3x - 7 > 5x - 1$ and show on the number line.

$3x - 5x > -1 + 7 \Rightarrow -2x > 6 \Rightarrow x < -3$ (sign reversed). Solution: **$(-\infty, -3)$** ; open circle at -3 , shade to the left.

Example 2. Solve $(3x - 4)/2 \geq (x + 1)/4 - 1$.

Multiply by 4: $2(3x - 4) \geq x + 1 - 4 \Rightarrow 6x - 8 \geq x - 3 \Rightarrow 5x \geq 5 \Rightarrow x \geq 1$, i.e. **$[1, \infty)$** .

Example 3. Solve $-8 \leq 5x - 3 < 7$.

Add 3: $-5 \leq 5x < 10$. Divide by 5: $-1 \leq x < 2$. Solution **$[-1, 2)$** .

Example 4. Ravi got 70 and 75 in two tests. What must he get in the third test so that his average is at least 60?

$(70 + 75 + x)/3 \geq 60 \Rightarrow 145 + x \geq 180 \Rightarrow x \geq 35$. He needs **at least 35 marks**.

Example 5. Find all pairs of consecutive odd natural numbers, both larger than 10, whose sum is less than 40.

Let them be x and $x + 2$. $x > 10$ and $x + x + 2 < 40 \Rightarrow x < 19$. So $x = 11, 13, 15, 17$. Pairs: **$(11,13), (13,15), (15,17), (17,19)$** .

XI-6. Permutations and Combinations

Idea: This chapter is about smart counting. **Permutation** = arrangement (order matters, like seating people in chairs). **Combination** = selection (order does not matter, like choosing a team).

FUNDAMENTAL PRINCIPLE OF COUNTING

If one job can be done in m ways and after that another job in n ways, both together can be done in $m \times n$ ways. (Example: 3 shirts and 4 trousers give $3 \times 4 = 12$ outfits.)

FORMULAS

$n! = 1 \times 2 \times 3 \times \dots \times n$, and $0! = 1$

${}^n P_r = n!/(n-r)!$ (arrangements of r things out of n)

${}^n C_r = n!/[r!(n-r)!]$ (selections of r things out of n)

${}^n P_r = r! \times {}^n C_r$ ${}^n C_r = {}^n C_{n-r}$ ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$

Arrangements of n things where p are alike, q are alike, ...: $n!/(p! q! \dots)$

If ${}^n C_a = {}^n C_b$ then $a = b$ or $a + b = n$

HOW TO DECIDE P OR C?

Ask: "If I swap two chosen items, do I get something new?" Yes $\rightarrow$ Permutation. No $\rightarrow$ Combination. Words like *arrange*, *words*, *numbers*, *seating* $\rightarrow$ P. Words like *select*, *choose*, *committee*, *team* $\rightarrow$ C.

Example 1. How many 3-digit numbers can be formed from digits 1, 2, 3, 4, 5 if (a) repetition is allowed (b) not allowed?

(a) $5 \times 5 \times 5 = 125$. (b) $5 \times 4 \times 3 = 60$.

Example 2. Find the number of arrangements of the letters of the word INDEPENDENCE.

12 letters: N appears 3 times, E appears 4 times, D appears 2 times. Number = $12!/(3! 4! 2!) = 479001600/(6 \times 24 \times 2) = 1663200$.

Example 3. In how many ways can the letters of the word PERMUTATIONS be arranged if all vowels come together?

Letters: 12, with T twice. Vowels E, U, A, I, O (5). Treat vowels as one block: 7 consonants + 1 block = 8 units, with T repeated: $8!/2! = 20160$. Vowels among themselves: $5! = 120$. Total = $20160 \times 120 = 2419200$.

Example 4. If ${}^n C_8 = {}^n C_2$, find ${}^n C_2$.

$8 \neq 2$ so $8 + 2 = n \Rightarrow n = 10$. ${}^{10} C_2 = 10 \times 9/2 = 45$.

Example 5. A committee of 3 men and 2 women is to be chosen from 7 men and 5 women. In how many ways?

${}^7 C_3 \times {}^5 C_2 = 35 \times 10 = 350$.

Example 6. How many chords can be drawn through 21 points on a circle?

A chord needs 2 points, order doesn't matter: ${}^{21} C_2 = 210$.

Example 7. In how many ways can a team of 11 be chosen from 17 players if 2 particular players must always be included?

Choose remaining 9 from 15: ${}^{15}C_9 = {}^{15}C_6 = 5005$.

Example 8. Find n if ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$.

${}^nP_4 = n \times {}^{n-1}P_3$. So ${}^{n-1}P_3 / {}^nP_4 = 1/n = 1/9 \Rightarrow n = 9$.

XI-7. Binomial Theorem

History: Indian mathematician Pingala (around 200 BC) described the pattern of coefficients now called Pascal's triangle ("Meru Prastara"). Blaise Pascal studied it later in Europe and Isaac Newton extended it.

BINOMIAL THEOREM FOR POSITIVE INTEGER N

$$(a + b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_n b^n$$

Number of terms = $n + 1$ Sum of coefficients = 2^n

General term: $T_{r+1} = {}^nC_r a^{n-r} b^r$

Middle term: if n is even $\rightarrow (n/2 + 1)$ th term; if n is odd $\rightarrow$ two middle terms, $((n+1)/2)$ th and $((n+3)/2)$ th

Pascal's triangle: each number is the sum of the two numbers above it.

```
n=0:      1
n=1:     1  1
n=2:    1  2  1
n=3:   1  3  3  1
n=4:  1  4  6  4  1
n=5: 1  5 10 10  5  1
```

Proof idea (by induction): The result is true for $n = 1$. Assume it is true for $n = k$. Multiply $(a + b)^k$ by $(a + b)$ and combine terms using ${}^kC_r + {}^kC_{r-1} = {}^{k+1}C_r$. The result is then true for $n = k + 1$, so by induction it holds for all positive integers.

Example 1. Expand $(2x - 3)^4$.

$$= (2x)^4 - 4(2x)^3(3) + 6(2x)^2(9) - 4(2x)(27) + 81 = 16x^4 - 96x^3 + 216x^2 - 216x + 81.$$

Example 2. Using the binomial theorem, evaluate $(99)^5$.

$$(100 - 1)^5 = 10^5 - 5 \cdot 10^8 + 10 \cdot 10^6 - 10 \cdot 10^4 + 5 \cdot 10^2 - 1 = 10000000000 - 500000000 + 10000000 - 10000 + 500 - 1 = 9509900499.$$

Example 3. Which is larger, $(1.01)^{1000000}$ or 10000?

$(1 + 0.01)^{1000000} = 1 + 1000000(0.01) + (\text{positive terms}) = 1 + 10000 + \dots > 10000$.
So **$(1.01)^{1000000}$ is larger.**

Example 4. Find the coefficient of x^5 in $(x + 3)^8$.

$T_{r+1} = {}^8C_r x^{8-r} 3^r$. $8 - r = 5 \Rightarrow r = 3$. Coefficient = ${}^8C_3 \times 27 = 56 \times 27 = \mathbf{1512}$.

Example 5. Find the middle term in $(3 - x^3/6)^7$.

$n = 7$ (odd), so middle terms are 4th and 5th. $T_4 = {}^7C_3 \cdot 3^4 \cdot (-x^3/6)^3 = 35 \times 81 \times (-x^9/216) = -105x^9/8$. $T_5 = {}^7C_4 \cdot 3^3 \cdot (x^{12}/1296) = 35 \times 27 x^{12}/1296 = 35x^{12}/48$.

Example 6. Find the term independent of x in $(x - 1/x)^6$.

$T_{r+1} = {}^6C_r x^{6-r} (-1)^r x^{-r} = {}^6C_r (-1)^r x^{6-2r}$. $6 - 2r = 0 \Rightarrow r = 3$. Term = ${}^6C_3 (-1)^3 = \mathbf{-20}$.

XI-8. Sequences and Series

Idea: A sequence is a list of numbers that follow a rule (2, 4, 6, 8, ...). A series is the sum of a sequence (2 + 4 + 6 + 8 + ...).

ARITHMETIC PROGRESSION (A.P.) - COMMON DIFFERENCE D

n-th term: $a_n = a + (n - 1)d$ Sum: $S_n = (n/2)[2a + (n - 1)d] = (n/2)(a + l)$

Arithmetic mean of a and b: $A = (a + b)/2$

GEOMETRIC PROGRESSION (G.P.) - COMMON RATIO R

n-th term: $a_n = ar^{n-1}$

Sum of n terms: $S_n = a(r^n - 1)/(r - 1)$ if $r > 1$; $= a(1 - r^n)/(1 - r)$ if $r < 1$

Infinite G.P. sum (only when $|r| < 1$): $S_\infty = a/(1 - r)$

Geometric mean of a and b: $G = \sqrt{ab}$

Relation: $A \geq G$ (A.M. is always $\geq$ G.M. for positive numbers); a, b are roots of $x^2 - 2Ax + G^2 = 0$

Example 1. Find the 10th term and the sum of 10 terms of the G.P. 5, 25, 125, ...

$a = 5, r = 5$. $a_{10} = 5 \times 5^9 = 5^{10} = \mathbf{9765625}$. $S_{10} = 5(5^{10} - 1)/4 = 5 \times 9765624/4 = \mathbf{12207030}$.

Example 2. Which term of the G.P. 2, $2\sqrt{2}$, 4, ... is 128?

$a = 2, r = \sqrt{2}$. $2(\sqrt{2})^{n-1} = 128 \Rightarrow (\sqrt{2})^{n-1} = 64 = (\sqrt{2})^{12} \Rightarrow n - 1 = 12 \Rightarrow \mathbf{n = 13}$.

Example 3. Find the sum of the infinite G.P. $1 + 1/3 + 1/9 + \dots$

$a = 1, r = 1/3$. $S_\infty = 1/(1 - 1/3) = \mathbf{3/2}$.

Example 4. Insert 3 numbers between 1 and 256 so that the resulting sequence is a G.P.

5 terms: 1, ?, ?, ?, 256. $ar^4 = 256 \Rightarrow r^4 = 256 \Rightarrow r = 4$. Numbers: **4, 16, 64** (or -4, 16, -64 for $r = -4$).

Example 5. Find the A.M. and G.M. of 4 and 16, and verify $A \geq G$.

$A = 20/2 = 10$; $G = \sqrt{64} = 8$. $10 \geq 8$ ✓.

Example 6. Express 0.333... as a fraction using an infinite G.P.

$0.3 + 0.03 + 0.003 + \dots$: $a = 3/10$, $r = 1/10$. $S = (3/10)/(9/10) = \mathbf{1/3}$.

Example 7. A man saves Rs 100 in the first month, and each month saves double the previous month. How much does he save in 10 months?

G.P.: $a = 100$, $r = 2$. $S_{10} = 100(2^{10} - 1)/(2 - 1) = 100 \times 1023 = \mathbf{Rs\ 102300}$.

Exam Q&A - Algebra

Q1. Find the conjugate and modulus of $z = -3 + 4i$. [2]

$\bar{z} = -3 - 4i$; $|z| = \sqrt{9 + 16} = 5$.

Q2. Find the value of $i + i^2 + i^3 + i^4$. [1]

$i - 1 - i + 1 = 0$.

Q3. Evaluate $8!/(6! \times 2!)$. [1]

$= (8 \times 7)/2 = 28$.

Q4. How many words (with or without meaning) can be made from the letters of MONDAY using 4 letters at a time with no repetition? [2]

${}^6P_4 = 6 \times 5 \times 4 \times 3 = 360$.

Q5. Solve $2x + 3 \leq 7$, $x \in \mathbf{N}$. [1]

$x \leq 2$, so $x \in \{1, 2\}$.

Q6. Find three numbers in G.P. whose sum is 13 and product is 27. [4]

Take the numbers as a/r , a , ar . Product $= a^3 = 27 \Rightarrow a = 3$. Sum: $3/r + 3 + 3r = 13 \Rightarrow 3r^2 - 10r + 3 = 0 \Rightarrow (3r - 1)(r - 3) = 0 \Rightarrow r = 3$ or $1/3$. Numbers: 1, 3, 9 (or 9, 3, 1).

Q7. State the relation between A.M. and G.M. and prove it for two positive numbers. [4]

For positive a , b : $A - G = (a + b)/2 - \sqrt{ab} = (\sqrt{a} - \sqrt{b})^2/2 \geq 0$. Hence $A \geq G$, with equality only when $a = b$.

XI-9. Straight Lines

Idea: In coordinate geometry we describe shapes using equations. A straight line is the simplest shape, and every straight line has an equation of the form $ax + by + c = 0$.

Recall from earlier classes

BASIC FORMULAS

Distance between (x_1, y_1) and (x_2, y_2) : $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Section formula (ratio $m : n$, internal): $(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n})$

Mid-point: $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

Area of triangle = $\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

Slope of a line

The slope (gradient) m tells how steep a line is: "rise over run". If a line makes angle θ with the positive x-axis, $m = \tan \theta$.

SLOPE FACTS

Slope through two points: $m = (y_2 - y_1)/(x_2 - x_1)$

Parallel lines: $m_1 = m_2$ Perpendicular lines: $m_1 m_2 = -1$

Angle between two lines: $\tan \theta = |(m_2 - m_1)/(1 + m_1 m_2)|$

Three points A, B, C are collinear if slope AB = slope BC

x-axis has slope 0; y-axis (vertical lines) slope is not defined

Forms of the equation of a line

Form	Equation	When to use
Parallel to x-axis	$y = b$	Horizontal line at height b
Parallel to y-axis	$x = a$	Vertical line
Point-slope	$y - y_1 = m(x - x_1)$	One point and slope known
Two-point	$y - y_1 = [(y_2 - y_1)/(x_2 - x_1)](x - x_1)$	Two points known
Slope-intercept	$y = mx + c$	Slope and y-intercept c known
Intercept	$x/a + y/b = 1$	x-intercept a and y-intercept b known
General	$Ax + By + C = 0$	Slope = $-A/B$, x-int = $-C/A$, y-int = $-C/B$

DISTANCE FORMULAS

Distance of point (x_1, y_1) from line $Ax + By + C = 0$: $d = |Ax_1 + By_1 + C| / \sqrt{A^2 + B^2}$

Distance between parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$: $d = |C_1 - C_2| / \sqrt{A^2 + B^2}$

Example 1. Find the slope of the line through $(3, -2)$ and $(-1, 4)$.

$m = (4 - (-2))/(-1 - 3) = 6/(-4) = -3/2$.

Example 2. Show that A(4, 4), B(3, 5), C(−1, −1) are vertices of a right-angled triangle.

Slope AB = $(5 - 4)/(3 - 4) = -1$. Slope AC = $(-1 - 4)/(-1 - 4) = 1$. Product = -1 , so $AB \perp AC$: right angle at A.

Example 3. Find the angle between lines with slopes $1/\sqrt{3}$ and $\sqrt{3}$.

$\tan \theta = |(\sqrt{3} - 1/\sqrt{3})/(1 + 1)| = |(2/\sqrt{3})/2| = 1/\sqrt{3} \Rightarrow \theta = 30^\circ$.

Example 4. Find the equation of the line through (−2, 3) with slope −4.

$y - 3 = -4(x + 2) \Rightarrow y - 3 = -4x - 8 \Rightarrow 4x + y + 5 = 0$.

Example 5. Find the equation of the line through (1, −1) and (3, 5).

Slope = $6/2 = 3$. $y + 1 = 3(x - 1) \Rightarrow 3x - y - 4 = 0$.

Example 6. Find the line which cuts off intercepts 3 and −2 on the axes.

$x/3 + y/(-2) = 1 \Rightarrow$ multiply by 6: $2x - 3y = 6 \Rightarrow 2x - 3y - 6 = 0$.

Example 7. Find the distance of (3, −5) from the line $3x - 4y - 26 = 0$.

$d = |3(3) - 4(-5) - 26|/\sqrt{9 + 16} = |9 + 20 - 26|/5 = 3/5$ units.

Example 8. Find the distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$.

$d = |7 - 5|/\sqrt{9 + 16} = 2/5$ units.

Example 9. Find the equation of the line through (2, 3) perpendicular to $3x + 4y - 5 = 0$.

Slope of given line = $-3/4$, so required slope = $4/3$. $y - 3 = (4/3)(x - 2) \Rightarrow 3y - 9 = 4x - 8 \Rightarrow 4x - 3y + 1 = 0$.

XI-10. Conic Sections

Idea: If you cut a double cone (two ice-cream cones joined at the tip) with a flat plane at different angles, you get different curves: **circle**, **ellipse**, **parabola**, **hyperbola**. These are called conic sections. If the plane passes through the tip (vertex), you get a **point**, a **straight line** or a **pair of intersecting lines** — these are "degenerate" conics.

Plane cuts the cone...	Curve obtained	Real-life example
perpendicular to the axis	Circle	Wheel, ring
slightly tilted	Ellipse	Orbits of planets
parallel to a slant side	Parabola	Path of a thrown ball, dish antenna
steeper, cuts both cones	Hyperbola	Cooling towers' shape

Circle

CIRCLE

Centre (h, k), radius r: $(x - h)^2 + (y - k)^2 = r^2$

Centre origin: $x^2 + y^2 = r^2$

General form $x^2 + y^2 + 2gx + 2fy + c = 0$: centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$

Parabola

A parabola is the set of points whose distance from a fixed point (the **focus**) equals their distance from a fixed line (the **directrix**).

Equation	Opens	Focus	Directrix	Axis	Latus rectum
$y^2 = 4ax$	Right	(a, 0)	$x = -a$	$y = 0$	4a
$y^2 = -4ax$	Left	(-a, 0)	$x = a$	$y = 0$	4a
$x^2 = 4ay$	Up	(0, a)	$y = -a$	$x = 0$	4a
$x^2 = -4ay$	Down	(0, -a)	$y = a$	$x = 0$	4a

Ellipse

An ellipse is the set of points the **sum** of whose distances from two fixed points (foci) is constant ($= 2a$). Think of a stretched circle.

ELLIPSE $x^2/A^2 + y^2/B^2 = 1$ ($A > B$)

$c^2 = a^2 - b^2$ Foci $(\pm c, 0)$ Vertices $(\pm a, 0)$

Major axis = 2a, Minor axis = 2b Eccentricity $e = c/a$ (< 1)

Latus rectum = $2b^2/a$

If the bigger number is under y^2 ($x^2/b^2 + y^2/a^2 = 1$), the major axis is along the y-axis: foci $(0, \pm c)$

Hyperbola

A hyperbola is the set of points the **difference** of whose distances from two foci is constant ($= 2a$).

HYPERBOLA $x^2/A^2 - y^2/B^2 = 1$

$c^2 = a^2 + b^2$ Foci $(\pm c, 0)$ Vertices $(\pm a, 0)$

Transverse axis = 2a, Conjugate axis = 2b $e = c/a$ (> 1)

Latus rectum = $2b^2/a$ For $y^2/a^2 - x^2/b^2 = 1$, foci are $(0, \pm c)$

MEMORY TRICK

Ellipse: **minus** inside c ($c^2 = a^2 - b^2$), $e < 1$. Hyperbola: **plus** inside c ($c^2 = a^2 + b^2$), $e > 1$. Circle: $e = 0$. Parabola: $e = 1$.

Example 1. Find the centre and radius of $x^2 + y^2 - 4x - 8y - 45 = 0$.

$2g = -4 \Rightarrow g = -2$; $2f = -8 \Rightarrow f = -4$; $c = -45$. Centre (2, 4). $r = \sqrt{(4 + 16 + 45)} = \sqrt{65}$.

Example 2. Find the equation of the circle with centre $(-3, 2)$ and radius 4.

$$(x + 3)^2 + (y - 2)^2 = 16 \Rightarrow x^2 + y^2 + 6x - 4y - 3 = 0.$$

Example 3. Find the circle passing through $(4, 1)$ and $(6, 5)$ with centre on the line $4x + y = 16$.

Let centre (h, k) . Equal distances: $(4 - h)^2 + (1 - k)^2 = (6 - h)^2 + (5 - k)^2 \Rightarrow 17 - 8h - 2k = 61 - 12h - 10k \Rightarrow 4h + 8k = 44 \Rightarrow h + 2k = 11$. With $4h + k = 16$: solving, $h = 3, k = 4$. $r^2 = 1 + 9 = 10$. Circle: $(x - 3)^2 + (y - 4)^2 = 10$.

Example 4. For $y^2 = 12x$ find focus, axis, directrix and latus rectum.

$4a = 12 \Rightarrow a = 3$. Focus $(3, 0)$, axis: x-axis, directrix $x = -3$, latus rectum $= 12$.

Example 5. Find the equation of the parabola with focus $(0, -3)$ and directrix $y = 3$.

Focus on negative y-axis $\Rightarrow x^2 = -4ay$ with $a = 3$: $x^2 = -12y$.

Example 6. For the ellipse $x^2/36 + y^2/16 = 1$, find the foci, vertices, axes, eccentricity and latus rectum.

$a = 6, b = 4$. $c = \sqrt{36 - 16} = \sqrt{20} = 2\sqrt{5}$. Foci $(\pm 2\sqrt{5}, 0)$; vertices $(\pm 6, 0)$; major axis 12; minor axis 8; $e = \sqrt{5}/3$; latus rectum $= 2(16)/6 = 16/3$.

Example 7. Find the equation of the ellipse with vertices $(\pm 13, 0)$ and foci $(\pm 5, 0)$.

$a = 13, c = 5 \Rightarrow b^2 = 169 - 25 = 144$. $x^2/169 + y^2/144 = 1$.

Example 8. For the hyperbola $9y^2 - 4x^2 = 36$, find the foci, vertices, eccentricity and latus rectum.

Divide by 36: $y^2/4 - x^2/9 = 1$. $a = 2, b = 3, c = \sqrt{13}$. Foci $(0, \pm\sqrt{13})$; vertices $(0, \pm 2)$; $e = \sqrt{13}/2$; latus rectum $= 2(9)/2 = 9$.

Example 9. Find the hyperbola with foci $(0, \pm 12)$ and latus rectum 36.

$c = 12, 2b^2/a = 36 \Rightarrow b^2 = 18a$. $a^2 + 18a = 144 \Rightarrow (a + 24)(a - 6) = 0 \Rightarrow a = 6, b^2 = 108$. $y^2/36 - x^2/108 = 1$.

XI-11. Introduction to Three-Dimensional Geometry

Idea: In real life we need three numbers to locate a point — length, breadth and height. A fly in a room is located by its distance from two walls and the floor.

KEY IDEAS

- Three mutually perpendicular axes: x-axis, y-axis, z-axis, meeting at origin $O(0, 0, 0)$.
- Three coordinate planes: XY-plane ($z = 0$), YZ-plane ($x = 0$), ZX-plane ($y = 0$).
- These planes divide space into **8 octants**.
- Any point on the x-axis is $(x, 0, 0)$; on the y-axis $(0, y, 0)$; on the z-axis $(0, 0, z)$.

Octant	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

DISTANCE FORMULA IN 3-D

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\text{Distance from origin: } \sqrt{x^2 + y^2 + z^2}$$

Example 1. Name the octant in which $(-3, 1, -2)$ lies.

Signs $(-, +, -) \Rightarrow$ **Octant VI**.

Example 2. Find the distance between $P(1, -3, 4)$ and $Q(-4, 1, 2)$.

$$PQ = \sqrt{(25 + 16 + 4)} = \sqrt{45} = 3\sqrt{5} \text{ units.}$$

Example 3. Show that $(-2, 3, 5)$, $(1, 2, 3)$ and $(7, 0, -1)$ are collinear.

$$AB = \sqrt{(9 + 1 + 4)} = \sqrt{14}; BC = \sqrt{(36 + 4 + 16)} = \sqrt{56} = 2\sqrt{14}; AC = \sqrt{(81 + 9 + 36)} = \sqrt{126} = 3\sqrt{14}. AB + BC = 3\sqrt{14} = AC, \text{ so the points are collinear.}$$

Example 4. Find the point on the y-axis equidistant from $(3, 1, 2)$ and $(5, 5, 2)$.

$$\text{Point } (0, y, 0). 9 + (y - 1)^2 + 4 = 25 + (y - 5)^2 + 4 \Rightarrow 13 + y^2 - 2y + 1 = 29 + y^2 - 10y + 25 \Rightarrow 8y = 40 \Rightarrow y = 5. \text{ Point } \mathbf{(0, 5, 0)}.$$

Exam Q&A - Coordinate Geometry

Q1. Find the slope of the line $3x - 4y + 10 = 0$. [1]

$$m = -A/B = -3/(-4) = 3/4.$$

Q2. Find the eccentricity of the parabola. [1]

$$e = 1.$$

Q3. Write the equation of the circle with centre $(0, 0)$ passing through $(3, 4)$. [1]

$$r = 5, \text{ so } x^2 + y^2 = 25.$$

Q4. Find the equation of the line whose perpendicular distance from the origin is 5 and the perpendicular makes 30° with the x-axis. [2]

$$\text{Use the normal form } x \cos \omega + y \sin \omega = p: x(\sqrt{3}/2) + y(1/2) = 5 \Rightarrow \sqrt{3}x + y = 10.$$

Q5. Find the length of the major axis of $4x^2 + 9y^2 = 36$. [2]

$$x^2/9 + y^2/4 = 1, a = 3, \text{ major axis} = 6.$$

Q6. What are the coordinates of the mid-point of the line joining (2, 3, 4) and (6, 7, 8)?

[1]

(4, 5, 6).

XI-12. Limits and Derivatives

Idea in simple words: Calculus is the mathematics of change. If a car travels and you want its speed at *exactly* 10 seconds (not the average over a minute), you need the idea of a **limit** — what value something approaches as you get closer and closer. The **derivative** is the instantaneous rate of change: speed is the derivative of distance.

Limits - intuitive idea

$\lim_{x \rightarrow a} f(x) = L$ means: as x comes very close to a (from both sides), $f(x)$ comes very close to L . The function need not even be defined at a .

Example: $f(x) = (x^2 - 1)/(x - 1)$ is not defined at $x = 1$, but for x close to 1, $f(x) = x + 1$ is close to 2. So $\lim_{x \rightarrow 1} f(x) = 2$.

LEFT AND RIGHT LIMITS

The left-hand limit ($x \rightarrow a^-$) and right-hand limit ($x \rightarrow a^+$) must be **equal** for the limit to exist.

STANDARD LIMITS

$$\lim_{x \rightarrow a} (x^n - a^n)/(x - a) = na^{n-1}$$

$$\lim_{x \rightarrow 0} (\sin x)/x = 1 \quad \lim_{x \rightarrow 0} (\tan x)/x = 1 \quad \lim_{x \rightarrow 0} (1 - \cos x)/x = 0$$

$$\lim_{x \rightarrow 0} (e^x - 1)/x = 1 \quad \lim_{x \rightarrow 0} (a^x - 1)/x = \log_e a \quad \lim_{x \rightarrow 0} \log(1 + x)/x = 1$$

$$\text{Algebra: } \lim(f \pm g) = \lim f \pm \lim g; \lim(fg) = \lim f \cdot \lim g; \lim(f/g) = \lim f / \lim g \text{ (if } \lim g \neq 0)$$

METHOD FOR ANY LIMIT QUESTION

1. Substitute $x = a$ directly. If you get a number, that is the answer.
2. If you get $0/0$, factorise and cancel, or rationalise, or use a standard limit.

Example 1. Evaluate $\lim_{x \rightarrow 3} (x^2 - 9)/(x - 3)$.

$0/0$ form. Factorise: $(x - 3)(x + 3)/(x - 3) = x + 3 \rightarrow 6$.

Example 2. Evaluate $\lim_{x \rightarrow 1} (x^{15} - 1)/(x^{10} - 1)$.

Divide top and bottom by $(x - 1)$: $[15 \cdot 1^{14}]/[10 \cdot 1^9] = 3/2$.

Example 3. Evaluate $\lim_{x \rightarrow 0} \sin 4x / \sin 2x$.

$= [\sin 4x/4x \cdot 4x] / [\sin 2x/2x \cdot 2x] = 4x/2x = 2$.

Example 4. Evaluate $\lim_{x \rightarrow 0} [\sqrt{(1 + x)} - 1]/x$.

Rationalise: $(1 + x - 1)/[x(\sqrt{(1 + x)} + 1)] = 1/(\sqrt{(1 + x)} + 1) \rightarrow 1/2$.

Example 5. Evaluate $\lim_{x \rightarrow 0} (1 - \cos 2x)/x^2$.

$$1 - \cos 2x = 2\sin^2 x. \text{ So } 2(\sin x/x)^2 \rightarrow 2.$$

Example 6. Evaluate $\lim_{x \rightarrow 0} (e^{3x} - 1)/x$.

$$= 3 \times (e^{3x} - 1)/(3x) \rightarrow 3.$$

Derivatives

DEFINITION (FIRST PRINCIPLE)

$$f'(x) = \lim_{h \rightarrow 0} [f(x+h) - f(x)]/h.$$

Geometrically, $f'(a)$ is the **slope of the tangent** to the curve $y = f(x)$ at $x = a$.

Physically, if $s(t)$ is distance, $s'(t)$ is velocity.

RULES AND STANDARD DERIVATIVES

$$d/dx (c) = 0 \quad d/dx (x^n) = nx^{n-1} \quad d/dx (cf) = c f'$$

$$\text{Sum/difference: } (u \pm v)' = u' \pm v'$$

$$\text{Product rule: } (uv)' = u'v + uv'$$

$$\text{Quotient rule: } (u/v)' = (u'v - uv')/v^2$$

$$d/dx \sin x = \cos x \quad d/dx \cos x = -\sin x \quad d/dx \tan x = \sec^2 x$$

$$d/dx \cot x = -\operatorname{cosec}^2 x \quad d/dx \sec x = \sec x \tan x \quad d/dx \operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

Example 7. Find the derivative of $f(x) = x^2$ from first principles.

$$f'(x) = \lim_{h \rightarrow 0} [(x+h)^2 - x^2]/h = \lim (2xh + h^2)/h = \lim (2x + h) = 2x.$$

Example 8. Find the derivative of $\sin x$ from first principles.

$$[\sin(x+h) - \sin x]/h = 2\cos(x+h/2) \sin(h/2)/h = \cos(x+h/2) \cdot [\sin(h/2)/(h/2)] \rightarrow \cos x \cdot 1 = \cos x.$$

Example 9. Differentiate $y = 5x^4 - 3x^2 + 7x - 9$.

$$dy/dx = 20x^3 - 6x + 7.$$

Example 10. Differentiate $y = x^2 \sin x$.

$$\text{Product rule: } 2x \sin x + x^2 \cos x.$$

Example 11. Differentiate $y = (x+1)/(x-1)$.

$$\text{Quotient rule: } [(1)(x-1) - (x+1)(1)]/(x-1)^2 = -2/(x-1)^2.$$

Example 12. Find the slope of the tangent to $y = x^3 - x$ at $x = 2$.

$$dy/dx = 3x^2 - 1 = 12 - 1 = 11.$$

Example 13. Differentiate $y = (\sin x + \cos x)/(\sin x - \cos x)$.

$$u'v - uv' = (\cos x - \sin x)(\sin x - \cos x) - (\sin x + \cos x)(\cos x + \sin x) = -(\sin x - \cos x)^2 - (\sin x + \cos x)^2 = -2. \text{ So } dy/dx = -2/(\sin x - \cos x)^2.$$

XI-13. Statistics (Measures of Dispersion)

Idea: Two classes may both have an average of 50 marks, but in one class everyone scored 45–55 while in the other marks range from 5 to 95. **Dispersion** measures how spread out the data is around the average.

FORMULAS

Range = Largest value – Smallest value

Mean deviation about mean (ungrouped): $M.D. = \sum |x_i - \bar{x}| / n$

Mean deviation (grouped): $M.D. = \sum f_i |x_i - \bar{x}| / N$, where $N = \sum f_i$

Mean deviation about median: replace $\bar{x}$ by M

Variance $\sigma^2 = \sum (x_i - \bar{x})^2 / n = (\sum x_i^2)/n - \bar{x}^2$

Grouped: $\sigma^2 = \sum f_i (x_i - \bar{x})^2 / N$

Short-cut (step-deviation): $y_i = (x_i - A)/h$, $\sigma^2 = h^2 [\sum f_i y_i^2 / N - (\sum f_i y_i / N)^2]$

Standard deviation $\sigma = \sqrt{\text{variance}}$ Coefficient of variation $C.V. = (\sigma/\bar{x}) \times 100$

WHICH SERIES IS MORE CONSISTENT?

The one with the **lower coefficient of variation** is more consistent (less variable).

Example 1. Find the mean deviation about the mean for 6, 7, 10, 12, 13, 4, 8, 12.

Sum = 72, $n = 8$, $\bar{x} = 9$. $|x - 9|$: 3, 2, 1, 3, 4, 5, 1, 3; total = 22. $M.D. = 22/8 = 2.75$.

Example 2. Find the mean and variance of 6, 8, 10, 12, 14, 16, 18, 20, 22, 24.

$\bar{x} = 150/10 = 15$. Deviations: -9, -7, -5, -3, -1, 1, 3, 5, 7, 9; squares sum = $2(81 + 49 + 25 + 9 + 1) = 330$. Variance = $330/10 = 33$; $\sigma = \sqrt{33} \approx 5.74$.

Example 3. Find the mean and standard deviation of the following data.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

$N = 30$. $\sum fx = 12 + 40 + 99 + 85 + 80 + 72 + 32 = 420$. $\bar{x} = 14$.

$\sum f(x - 14)^2 = 3(100) + 5(36) + 9(9) + 5(9) + 4(36) + 3(100) + 1(324) = 300 + 180 + 81 + 45 + 144 + 300 + 324 = 1374$.

$\sigma^2 = 1374/30 = 45.8$; $\sigma = 6.77$ (approx).

Example 4. Two plants A and B: A has mean wage Rs 5000 with variance 81; B has mean Rs 5000 with variance 100. Which plant shows greater variability?

$\sigma_A = 9$, $\sigma_B = 10$. $C.V._A = 0.18\%$, $C.V._B = 0.2\%$. **Plant B** has greater variability.

Example 5. Mean deviation for grouped data (class intervals).

Marks	0-10	10-20	20-30	30-40	40-50
Students	5	8	15	16	6

Mid-points: 5, 15, 25, 35, 45. $N = 50$. $\Sigma fx = 25 + 120 + 375 + 560 + 270 = 1350$. $\bar{x} = 27$.

$|x - 27|$: 22, 12, 2, 8, 18. $\Sigma f|x - 27| = 110 + 96 + 30 + 128 + 108 = 472$. M.D. = $472/50 = 9.44$.

XI-14. Probability

Idea: Probability measures how likely something is, on a scale from 0 (impossible) to 1 (certain).

DEFINITIONS

- **Random experiment:** outcome cannot be predicted in advance (tossing a coin).
- **Sample space S:** set of all possible outcomes. Coin: {H, T}; die: {1,2,3,4,5,6}.
- **Event:** any subset of S. "Getting an even number" = {2, 4, 6}.
- **'Not A' event:** A' ; **'A and B':** $A \cap B$; **'A or B':** $A \cup B$.
- **Mutually exclusive:** cannot happen together, $A \cap B = \varnothing$.
- **Exhaustive events:** together they cover the whole sample space, $A \cup B \cup \dots = S$.

AXIOMATIC PROBABILITY AND RULES

Axioms: (i) $P(E) \geq 0$ (ii) $P(S) = 1$ (iii) for mutually exclusive A, B: $P(A \cup B) = P(A) + P(B)$

For equally likely outcomes: $P(A) = n(A)/n(S)$

$P(\text{not } A) = 1 - P(A)$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ but not } B) = P(A) - P(A \cap B)$

Example 1. A die is thrown. Find the probability of getting (a) a prime number (b) a number greater than 4.

S has 6 outcomes. Primes {2, 3, 5}: $P = 3/6 = 1/2$. Greater than 4 {5, 6}: $P = 1/3$.

Example 2. Two coins are tossed. Find the probability of getting at least one head.

$S = \{HH, HT, TH, TT\}$. At least one head = 3 outcomes. $P = 3/4$. (Or $1 - P(\text{no head}) = 1 - 1/4$.)

Example 3. Two dice are thrown. Find the probability that the sum is 8.

$n(S) = 36$. Sum 8: (2,6), (3,5), (4,4), (5,3), (6,2) = 5. $P = 5/36$.

Example 4. A card is drawn from a pack of 52. Find P(king or heart).

$P(K) = 4/52$, $P(H) = 13/52$, $P(K \text{ and } H) = 1/52$. $P = (4 + 13 - 1)/52 = 16/52 = 4/13$.

Example 5. If $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$, find $P(\text{not } A)$, $P(\text{not } B)$, $P(A \text{ or } B)$.

$$P(A') = 0.58, P(B') = 0.52, P(A \cup B) = 0.42 + 0.48 - 0.16 = \mathbf{0.74}.$$

Example 6. 4 cards are drawn from a pack of 52. Find the probability that all are spades.

$$P = {}^{13}C_4 / {}^{52}C_4 = 715/270725 = \mathbf{11/4165}.$$

Example 7. A committee of 2 is chosen from 2 men and 2 women. Find the probability that it has exactly one man.

$$\text{Total} = {}^4C_2 = 6. \text{ One man and one woman} = 2 \times 2 = 4. P = 4/6 = \mathbf{2/3}.$$

Exam Q&A - Calculus, Statistics, Probability

Q1. Evaluate $\lim_{x \rightarrow 2} (3x + 4)$. [1]

Substitute: $3(2) + 4 = 10$.

Q2. Differentiate $x^{-3}(5 + 3x)$. [2]

$$y = 5x^{-3} + 3x^{-2} \Rightarrow y' = -15x^{-4} - 6x^{-3}.$$

Q3. What is the variance of the data if every value is 7? [1]

Zero, because there is no spread.

Q4. If each observation is increased by 5, what happens to the mean and to the variance? [2]

The mean increases by 5; the variance stays the same (spread does not change).

Q5. Can $P(A) = 1.2$? [1]

No. Probability always lies between 0 and 1.

Q6. Are the events "getting an odd number" and "getting an even number" on a die mutually exclusive and exhaustive? [2]

Yes. They cannot happen together (exclusive), and together they make $\{1, \dots, 6\}$ (exhaustive).

CHAPTER-WISE QUESTION BANK

Mathematics · +2 1st Year · MCQs, short and long questions with full solutions

HOW TO USE THIS QUESTION BANK

Cover the solution, try the question yourself, then compare. Mark questions you got wrong and repeat them after a week. MCQ answers are given in brackets with a one-line reason so you learn the idea, not just the option.

QB-1. Sets

Key points to remember

- A set must be well-defined. Order and repetition of elements do not matter.
- $\varnothing$ is a subset of every set; every set is a subset of itself.
- If $n(A) = m$: number of subsets 2^m , proper subsets $2^m - 1$, $n(P(A)) = 2^m$.
- $A - B = A \cap B'$; $A \cup A' = U$; $A \cap A' = \varnothing$.
- $n(A - B) = n(A) - n(A \cap B)$; $n(\text{only } A) = n(A) - n(A \cap B)$.
- Distributive laws: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

A. Multiple choice questions

1. Which of the following is the empty set? (a) $\{0\}$ (b) $\{x : x \in \mathbb{N}, x^2 = 4\}$ (c) $\{x : x \text{ is an even prime greater than } 2\}$ (d) $\{\varnothing\}$

(c). The only even prime is 2, so there is no even prime greater than 2. $\{0\}$ and $\{\varnothing\}$ each have one element.

2. If $A = \{1, 2, 3\}$, the number of proper subsets of A is (a) 8 (b) 7 (c) 6 (d) 3

(b) $2^3 - 1 = 7$.

3. The set $\{x : x \in \mathbb{R}, 2 < x \leq 5\}$ in interval form is (a) $(2, 5)$ (b) $[2, 5]$ (c) $(2, 5]$ (d) $[2, 5)$

(c) 2 excluded (round bracket), 5 included (square bracket).

4. If $A \subset B$, then $A \cap B$ equals (a) A (b) B (c) ϕ (d) U

(a) Every element of A is already in B .

5. If $A \subset B$, then $A \cup B$ equals (a) A (b) B (c) ϕ (d) A'

(b).

6. $(A')'$ equals (a) A' (b) U (c) A (d) ϕ

(c) Complement of the complement is the set itself.

7. If $n(A) = 4$, $n(B) = 6$ and $n(A \cap B) = 2$, then $n(A \cup B) =$ (a) 10 (b) 8 (c) 12 (d) 6

(b) $4 + 6 - 2 = 8$.

8. Which set is infinite? (a) $\{x : x \text{ is a month of the year}\}$ (b) set of lines through a point (c) set of students in a class (d) $\{x : x \in \mathbb{Z}, -5 < x < 5\}$

(b) Infinitely many lines pass through a point.

9. The number of elements in the power set of $\{a, b, c, d\}$ is (a) 4 (b) 8 (c) 16 (d) 15

(c) $2^4 = 16$.

10. $A - B$ is equal to (a) $A \cap B$ (b) $A \cap B'$ (c) $A' \cap B$ (d) $A \cup B'$

(b) Elements in A but not in B .

11. $\{x : x \text{ is a letter of the word "MATHEMATICS"}\}$ has how many elements? (a) 11 (b) 8 (c) 9 (d) 7

(b) M, A, T, H, E, I, C, S = 8.

12. If $A = \{2, 4, 6\}$ and $B = \{4, 6, 8\}$, then $A \Delta B = (A - B) \cup (B - A)$ is (a) $\{2, 8\}$ (b) $\{4, 6\}$ (c) $\{2, 4, 6, 8\}$ (d) ϕ

(a).

B. Very short answer questions (1 mark)

1. Write $\{x : x = 2n - 1, n \in \mathbb{N}, n \leq 5\}$ in roster form.

$\{1, 3, 5, 7, 9\}$.

2. Write $\{1, 4, 9, 16, 25\}$ in set-builder form.

$\{x : x = n^2, n \in \mathbb{N}, n \leq 5\}$.

3. Are $\{a, b\}$ and $\{b, a\}$ equal sets?

Yes — order does not matter.

4. Is $\{0\}$ an empty set?

No; it contains the element 0.

5. If $A = \{1, 2\}$, is $\{1\} \in P(A)$?

Yes; $\{1\}$ is a subset of A , so it is an element of $P(A)$.

6. Write $[-3, 0)$ in set-builder form.

$\{x : x \in \mathbb{R}, -3 \leq x < 0\}$.

7. If $U = \{1, 2, \dots, 10\}$ and $A = \{2, 4, 6, 8, 10\}$, find A' .

$\{1, 3, 5, 7, 9\}$.

8. State whether true or false: $\phi \subset \{a\}$.

True; ϕ is a subset of every set.

C. Short answer questions (2-4 marks)

1. Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$. Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

$$B \cap C = \{5, 6\}; A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6\}.$$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$; $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$; their intersection = $\{1, 2, 3, 4, 5, 6\}$. Both sides are equal. Verified.

2. If $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, b, c\}$, $B = \{b, d, f, h\}$, verify $(A \cap B)' = A' \cup B'$.

$$A \cap B = \{b\}; (A \cap B)' = \{a, c, d, e, f, g, h\}.$$

$$A' = \{d, e, f, g, h\}; B' = \{a, c, e, g\}; A' \cup B' = \{a, c, d, e, f, g, h\}. \text{ Equal. Verified.}$$

3. Show that $A \cap B = A$ implies $A \subset B$.

Let $x \in A$. Since $A = A \cap B$, $x \in A \cap B$, so $x \in B$. Thus every element of A is in B , i.e. $A \subset B$.

4. Prove that $A - (A - B) = A \cap B$.

$$A - (A - B) = A \cap (A \cap B')' = A \cap (A' \cup B) \text{ (De Morgan)} = (A \cap A') \cup (A \cap B) = \phi \cup (A \cap B) = A \cap B.$$

5. Draw a Venn diagram and shade $(A \cup B)'$ and $A \cap B'$.

Draw rectangle U with two overlapping circles A and B . $(A \cup B)'$: shade the region of the rectangle outside both circles. $A \cap B'$: shade only the part of circle A that does not overlap B (the "crescent" of A).

6. If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$.

$$38 = 17 + 23 - n(X \cap Y) \Rightarrow n(X \cap Y) = 2.$$

7. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many can speak both, if everyone speaks at least one?

$$400 = 250 + 200 - x \Rightarrow x = 50.$$

8. In a school, 20 teachers teach Maths or Physics. 12 teach Maths and 4 teach both. How many teach Physics? How many teach only Physics?

$$20 = 12 + P - 4 \Rightarrow P = 12. \text{ Only Physics} = 12 - 4 = 8.$$

D. Long answer questions (5-6 marks)

1. In a survey of 60 people, 25 read newspaper H, 26 read T, 26 read I, 9 read H and I, 11 read H and T, 8 read T and I, and 3 read all three. Find (i) the number who read at least one newspaper (ii) the number who read exactly one.

(i) $n(H \cup T \cup I) = 25 + 26 + 26 - 9 - 11 - 8 + 3 = 52$.

(ii) Exactly two: $(H \cap T \text{ only}) = 11 - 3 = 8$; $(H \cap I \text{ only}) = 9 - 3 = 6$; $(T \cap I \text{ only}) = 8 - 3 = 5$. Total exactly two = 19. All three = 3.

Exactly one = $52 - 19 - 3 = 30$.

2. A survey shows 63% of people like cheese and 76% like apples. If $x\%$ like both, show that $39 \leq x \leq 63$.

$n(C \cup A) \leq 100 \Rightarrow 63 + 76 - x \leq 100 \Rightarrow x \geq 39$. Also $C \cap A \subset C \Rightarrow x \leq 63$. Hence $39 \leq x \leq 63$.

3. Of 100 students, 15 passed in English, 12 in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science, and 4 in all three. Find how many passed (i) in English and Mathematics but not Science (ii) in Mathematics only (iii) in more than one subject (iv) in none.

(i) $6 - 4 = 2$.

(ii) Maths only = $12 - 6 - 7 + 4 = 3$.

(iii) Exactly two: $E \cap M$ only 2, $M \cap S$ only 3, $E \cap S$ only 0; plus all three 4 $\rightarrow 2 + 3 + 0 + 4 = 9$.

(iv) At least one = $15 + 12 + 8 - 6 - 7 - 4 + 4 = 22$; none = $100 - 22 = 78$.

QB-2. Relations and Functions**Key points to remember**

- $n(A \times B) = n(A) \times n(B)$; $A \times B \neq B \times A$ unless $A = B$ (or one is empty).
- Number of relations from A to B = $2^{n(A) \cdot n(B)}$.
- A function: every element of the domain has exactly one image.
- Domain of $\sqrt{f(x)}$: $f(x) \geq 0$. Domain of $1/f(x)$: $f(x) \neq 0$. Domain of $\log f(x)$: $f(x) > 0$.
- $[x]$ = greatest integer $\leq x$; $|x| \geq 0$; signum takes only $-1, 0, 1$.

A. Multiple choice questions

1. If $n(A) = 3$ and $n(B) = 2$, $n(A \times B)$ is (a) 5 (b) 6 (c) 8 (d) 9

(b) 3×2 .

2. Number of relations from a set with 2 elements to a set with 3 elements is (a) 6 (b) 32 (c) 64 (d) 8

(c) $2^6 = 64$.

3. The domain of $f(x) = 1/(x - 3)$ is (a) $\mathbb{R}$ (b) $\mathbb{R} - \{3\}$ (c) $\mathbb{R} - \{-3\}$ (d) $(3, \infty)$

(b).

4. The range of $f(x) = x^2$ ($x \in \mathbb{R}$) is (a) $\mathbb{R}$ (b) $[0, \infty)$ (c) $(0, \infty)$ (d) $(-\infty, 0]$

(b) Squares are never negative and 0 is attained.

5. $[-1.5]$ equals (a) -1 (b) -2 (c) 1 (d) -1.5

(b) Greatest integer not exceeding -1.5 is -2 .

6. The range of the signum function is (a) $\mathbb{R}$ (b) $\{-1, 1\}$ (c) $\{-1, 0, 1\}$ (d) $[-1, 1]$

(c).

7. Which relation is a function? (a) $\{(1,2), (1,3)\}$ (b) $\{(1,2), (2,2), (3,2)\}$ (c) $\{(2,1), (2,2)\}$ (d) $\{(1,1), (1,2), (2,1)\}$

(b) Each first element appears once.

8. If $f(x) = x^2 - 3x + 2$, then $f(-1) =$ (a) 0 (b) 6 (c) 2 (d) 4

(b) $1 + 3 + 2$.

9. The domain of $f(x) = \sqrt{4 - x^2}$ is (a) $[-2, 2]$ (b) $(-2, 2)$ (c) $[0, 2]$ (d) $\mathbb{R}$

(a) $4 - x^2 \geq 0 \Rightarrow -2 \leq x \leq 2$.

10. The range of $f(x) = |x - 2| + 3$ is (a) $[3, \infty)$ (b) $[2, \infty)$ (c) $\mathbb{R}$ (d) $(3, \infty)$

(a) Minimum 3 at $x = 2$.

B. Very short answer questions

1. If $A = \{a, b\}$ find $A \times A$.

$\{(a,a), (a,b), (b,a), (b,b)\}$.

2. If $(x/3 + 1, y - 2/3) = (5/3, 1/3)$, find x and y .

$x/3 = 2/3 \Rightarrow x = 2; y = 1$.

3. If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$, find A and B .

$A = \{a, b\}, B = \{x, y\}$.

4. Find $f(0)$ for $f(x) = (x^2 + 1)/(x + 1)$.

1.

5. Find the domain of $f(x) = \log(x - 5)$.

$x - 5 > 0 \Rightarrow (5, \infty)$.

C. Short answer questions

1. If $A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{4, 5\}$, verify $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

$B \cap C = \{4\}; A \times \{4\} = \{(1,4), (2,4)\}$. $A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$; $A \times C = \{(1,4), (1,5), (2,4), (2,5)\}$; intersection = $\{(1,4), (2,4)\}$. Equal.

2. Let R be the relation on $\mathbb{Z}$ defined by $R = \{(a, b) : a, b \in \mathbb{Z}, a - b \text{ is an integer}\}$. Find the domain and range.

For any integers a, b , $a - b$ is an integer, so every pair is in R . Domain = $\mathbb{Z}$, Range = $\mathbb{Z}$.

3. Let $A = \{1, 2, 3, 4, 6\}$. $R = \{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$. Write R , its domain and range.

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)\}$. Domain = $\{1, 2, 3, 4, 6\}$; Range = $\{1, 2, 3, 4, 6\}$.

4. Find the range of $f(x) = 1 - |x - 2|$.

$|x - 2| \geq 0 \Rightarrow -|x - 2| \leq 0 \Rightarrow f(x) \leq 1$. Range = $(-\infty, 1]$.

5. Find the domain and range of $f(x) = (x - 2)/(3 - x)$.

Domain: $x \neq 3$, i.e. $\mathbb{R} - \{3\}$. Range: let $y = (x - 2)/(3 - x) \Rightarrow 3y - xy = x - 2 \Rightarrow x(1 + y) = 3y + 2 \Rightarrow x = (3y + 2)/(1 + y)$, defined for $y \neq -1$. Range = $\mathbb{R} - \{-1\}$.

6. If $f(x) = x + 1/x$, prove that $[f(x)]^3 = f(x^3) + 3f(1/x)$.

$[f(x)]^3 = (x + 1/x)^3 = x^3 + 1/x^3 + 3(x + 1/x)$. $f(x^3) = x^3 + 1/x^3$; $f(1/x) = 1/x + x$. So
RHS = $x^3 + 1/x^3 + 3(x + 1/x) = \text{LHS}$.

D. Long answer question

Draw the graphs of (i) $f(x) = |x|$ (ii) $f(x) = [x]$ for $-2 \leq x < 3$ and write their domain and range. Also find $(f + g)(x)$ and $(f/g)(x)$ where $f(x) = \sqrt{x}$ and $g(x) = x$ ($x > 0$).

(i) $|x|$: V-shaped graph with vertex at origin, lines $y = x$ ($x \geq 0$) and $y = -x$ ($x < 0$).

Domain $\mathbb{R}$, range $[0, \infty)$.

(ii) $[x]$ on $[-2, 3)$: steps $-y = -2$ on $[-2, -1)$, -1 on $[-1, 0)$, 0 on $[0, 1)$, 1 on $[1, 2)$, 2 on $[2, 3)$. Each step has a filled dot at the left end and an open dot at the right. Domain $[-2, 3)$, range $\{-2, -1, 0, 1, 2\}$.

$(f + g)(x) = \sqrt{x} + x$; $(f/g)(x) = \sqrt{x}/x = 1/\sqrt{x}$, $x > 0$.

QB-3. Trigonometric Functions**Key points to remember**

- $180^\circ = \pi$ rad; $1^\circ = 60'$; $1' = 60''$.
- Allied angles: $\sin(\pi - x) = \sin x$, $\cos(\pi - x) = -\cos x$, $\sin(\pi + x) = -\sin x$, $\cos(\pi + x) = -\cos x$, $\sin(2\pi - x) = -\sin x$, $\cos(2\pi - x) = \cos x$, $\sin(\pi/2 - x) = \cos x$, $\cos(\pi/2 + x) = -\sin x$.
- Periods: $\sin, \cos, \sec, \csc - 2\pi$; $\tan, \cot - \pi$.
- $\sin(-x) = -\sin x$ (odd), $\cos(-x) = \cos x$ (even).
- Product to sum: $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$; $2 \cos A \cos B = \cos(A + B) + \cos(A - B)$; $2 \sin A \sin B = \cos(A - B) - \cos(A + B)$.

A. Multiple choice questions

1. The radian measure of 240° is (a) $2\pi/3$ (b) $4\pi/3$ (c) $5\pi/3$ (d) $3\pi/4$

(b) $240 \times \pi/180$.

2. The value of $\sin 120^\circ$ is (a) $1/2$ (b) $-1/2$ (c) $\sqrt{3}/2$ (d) $-\sqrt{3}/2$

(c) $\sin(180^\circ - 60^\circ) = \sin 60^\circ$.

3. The value of $\cos 210^\circ$ is (a) $\sqrt{3}/2$ (b) $-\sqrt{3}/2$ (c) $1/2$ (d) $-1/2$

(b) $\cos(180^\circ + 30^\circ) = -\cos 30^\circ$.

4. The maximum value of $3 \sin x + 4 \cos x$ is (a) 7 (b) 5 (c) 1 (d) 12

(b) $\sqrt{3^2 + 4^2}$.

5. $\sin 2x/(1 + \cos 2x)$ equals (a) $\cot x$ (b) $\tan x$ (c) $\sin x$ (d) $\tan 2x$

(b) $2 \sin x \cos x/(2\cos^2 x)$.

6. If $\tan A = 1/2$ and $\tan B = 1/3$, then $A + B =$ (a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/6$

(c) $\tan(A + B) = (5/6)/(1 - 1/6) = 1$.

7. $\cos^2 x - \sin^2 x$ equals (a) 1 (b) $\cos 2x$ (c) $\sin 2x$ (d) 0

(b).

8. In which quadrant is $\sec x$ negative and $\tan x$ positive? (a) I (b) II (c) III (d) IV

(c) $\cos$ negative and $\tan$ positive.

9. The period of $\sin 2x$ is (a) 2π (b) π (c) $\pi/2$ (d) 4π

(b) $2\pi/2$.

10. $2 \sin 15^\circ \cos 15^\circ =$ (a) $1/2$ (b) $\sqrt{3}/2$ (c) 1 (d) $1/\sqrt{2}$

(a) $\sin 30^\circ$.

B. Very short answer questions

1. Convert $5\pi/3$ radians to degrees.

300° .

2. Find $\cot(-15\pi/4)$.

$\cot$ is odd: $-\cot(15\pi/4) = -\cot(4\pi - \pi/4) = -(-\cot \pi/4) = 1$.

3. Find $\tan(19\pi/3)$.

$19\pi/3 = 6\pi + \pi/3 \Rightarrow \tan(\pi/3) = \sqrt{3}$.

4. Write the value of $\sin^2 25^\circ + \sin^2 65^\circ$.

$\sin 65^\circ = \cos 25^\circ \Rightarrow \text{sum} = 1$.

5. Find the range of $2 + 3 \cos x$.

$[-1, 5]$.

C. Short answer questions

1. Find the angle in radians through which a pendulum swings if its length is 75 cm and the tip describes an arc of 21 cm.

$\theta = l/r = 21/75 = 7/25$ radian.

2. Find the values of the other five ratios if $\sin x = 3/5$ and x lies in the second quadrant.

$\cos x = -4/5$; $\tan x = -3/4$; $\cot x = -4/3$; $\sec x = -5/4$; $\operatorname{cosec} x = 5/3$.

3. Prove: $\sin^2(\pi/6) + \cos^2(\pi/3) - \tan^2(\pi/4) = -1/2$.

$1/4 + 1/4 - 1 = -1/2$.

4. Prove: $2 \sin^2(3\pi/4) + 2 \cos^2(\pi/4) + 2 \sec^2(\pi/3) = 10$.

$2(1/2) + 2(1/2) + 2(4) = 1 + 1 + 8 = 10$.

5. Prove: $\cos(3\pi/2 + x) \cos(2\pi + x)[\cot(3\pi/2 - x) + \cot(2\pi + x)] = 1$.

$\cos(3\pi/2 + x) = \sin x$; $\cos(2\pi + x) = \cos x$; $\cot(3\pi/2 - x) = \tan x$; $\cot(2\pi + x) = \cot x$.

LHS = $\sin x \cos x (\tan x + \cot x) = \sin x \cos x \cdot 1/(\sin x \cos x) = 1$.

6. Prove: $(\cos 7x + \cos 5x)/(\sin 7x - \sin 5x) = \cot x$.

Numerator $2 \cos 6x \cos x$; denominator $2 \cos 6x \sin x$. Ratio = $\cot x$.

7. Prove: $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$.

LHS = $(\cos 4x/\sin 4x)(2 \sin 4x \cos x) = 2 \cos 4x \cos x$. RHS = $(\cos x/\sin x)(2 \cos 4x \sin x) = 2 \cos 4x \cos x$. Equal.

8. Prove: $\tan 4x = 4 \tan x(1 - \tan^2 x)/(1 - 6 \tan^2 x + \tan^4 x)$.

Let $t = \tan x$. $\tan 2x = 2t/(1 - t^2)$. $\tan 4x = 2 \tan 2x/(1 - \tan^2 2x) = [4t/(1 - t^2)]/[1 - 4t^2/(1 - t^2)^2] = 4t(1 - t^2)/[(1 - t^2)^2 - 4t^2] = 4t(1 - t^2)/(1 - 6t^2 + t^4)$.

D. Long answer questions**1. Prove: $\cos 2x \cos(x/2) - \cos 3x \cos(9x/2) = \sin 5x \sin(5x/2)$.**

$2 \times \text{LHS} = 2\cos 2x \cos(x/2) - 2\cos(9x/2)\cos 3x = [\cos(5x/2) + \cos(3x/2)] - [\cos(15x/2) + \cos(3x/2)] = \cos(5x/2) - \cos(15x/2) = -2 \sin(5x) \sin(-5x/2) = 2 \sin 5x \sin(5x/2)$.

Dividing by 2 gives LHS = $\sin 5x \sin(5x/2)$.

2. Prove: $\cos^2 x + \cos^2(x + \pi/3) + \cos^2(x - \pi/3) = 3/2$.

Use $\cos^2 \theta = (1 + \cos 2\theta)/2$. Sum = $3/2 + \frac{1}{2}[\cos 2x + \cos(2x + 2\pi/3) + \cos(2x - 2\pi/3)]$.

Now $\cos(2x + 2\pi/3) + \cos(2x - 2\pi/3) = 2\cos 2x \cos(2\pi/3) = 2\cos 2x(-1/2) = -\cos 2x$.

So the bracket = 0 and the sum = $3/2$.

3. Find $\sin(x/2)$, $\cos(x/2)$, $\tan(x/2)$ if $\cos x = -1/3$ and x lies in the third quadrant.

$\pi < x < 3\pi/2 \Rightarrow \pi/2 < x/2 < 3\pi/4$ (second quadrant: $\sin +$, $\cos -$).

$\sin^2(x/2) = (1 - \cos x)/2 = (4/3)/2 = 2/3 \Rightarrow \sin(x/2) = \sqrt{(2/3)} = \sqrt{6}/3$.

$\cos^2(x/2) = (1 + \cos x)/2 = 1/3 \Rightarrow \cos(x/2) = -1/\sqrt{3} = -\sqrt{3}/3$.

$\tan(x/2) = -\sqrt{2}$.

4. Prove: $\sin 3x + \sin 2x - \sin x = 4 \sin x \cos(x/2) \cos(3x/2)$.

$\sin 3x - \sin x = 2 \cos 2x \sin x$. So LHS = $2 \sin x \cos 2x + 2 \sin x \cos x = 2 \sin x (\cos 2x + \cos x) = 2 \sin x \cdot 2\cos(3x/2)\cos(x/2) = 4 \sin x \cos(x/2) \cos(3x/2)$.

QB-4. Complex Numbers and Quadratic Equations

Key points to remember

- $i^{4k} = 1, i^{4k+1} = i, i^{4k+2} = -1, i^{4k+3} = -i$. Sum of four consecutive powers of $i = 0$.
- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ is **not** true when both a and b are negative: $\sqrt{-4} \times \sqrt{-9} = 2i \times 3i = -6$, not $\sqrt{36}$.
- Two complex numbers $a + ib$ and $c + id$ are equal only if $a = c$ and $b = d$.
- $z + \bar{z} = 2\text{Re}(z)$; $z - \bar{z} = 2i \text{Im}(z)$; $z \bar{z} = |z|^2$.
- For a quadratic with real coefficients, complex roots occur in conjugate pairs.

A. Multiple choice questions

1. $i^{57} + 1/i^{25} =$ (a) 0 (b) $2i$ (c) $-2i$ (d) 2

(a) $i^{57} = i$; $1/i^{25} = 1/i = -i$. Sum 0.

2. The conjugate of $1/(3 + 4i)$ is (a) $(3 + 4i)/25$ (b) $(3 - 4i)/25$ (c) $3 + 4i$ (d) $3 - 4i$

(a) $1/(3 + 4i) = (3 - 4i)/25$, its conjugate is $(3 + 4i)/25$.

3. $|(1 + i)^4| =$ (a) 2 (b) 4 (c) $\sqrt{2}$ (d) 16

(b) $|1 + i|^4 = (\sqrt{2})^4$.

4. If $z = -i$, then $z^2 =$ (a) 1 (b) -1 (c) i (d) $-i$

(b).

5. The roots of $x^2 + 9 = 0$ are (a) ± 3 (b) $\pm 3i$ (c) $\pm 9i$ (d) $3i$ only

(b).

6. The point representing $2 - 3i$ in the Argand plane lies in (a) I (b) II (c) III (d) IV quadrant

(d) $(2, -3)$.

7. $(1 + i)^2 =$ (a) 2 (b) $2i$ (c) $-2i$ (d) 0

(b) $1 + 2i - 1$.

8. If $(x + iy)(2 - 3i) = 4 + i$, then $x =$ (a) $5/13$ (b) $14/13$ (c) $-5/13$ (d) 1

(a) $x + iy = (4 + i)/(2 - 3i) = (4 + i)(2 + 3i)/13 = (8 + 12i + 2i - 3)/13 = (5 + 14i)/13$.

B. Very short answer questions

1. Express $i^9 + i^{19}$ in $a + ib$ form.

$i + (-i) = 0 + 0i$.

2. Find the real and imaginary parts of $(2 - i)^2$.

$4 - 4i - 1 = 3 - 4i$: real 3, imaginary -4 .

3. Find x, y if $3x + (2x - y)i = 6 - 3i$.

$x = 2$; $4 - y = -3 \Rightarrow y = 7$.

4. Find the modulus of $(1 + i)/(1 - i)$.

$|1 + i|/|1 - i| = 1$.

C. Short answer questions

1. Express $(1/3 + 3i)^3$ in $a + ib$ form.

$(a + b)^3$ with $a = 1/3$, $b = 3i$: $1/27 + 3(1/9)(3i) + 3(1/3)(9i^2) + 27i^3 = 1/27 + i - 9 - 27i = -242/27 - 26i$.

2. Find the multiplicative inverse of $\sqrt{5} + 3i$.

$(\sqrt{5} - 3i)/(5 + 9) = \sqrt{5}/14 - (3/14)i$.

3. Express $(3 + i\sqrt{5})(3 - i\sqrt{5})/[(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})]$ in $a + ib$ form.

Numerator = $9 + 5 = 14$. Denominator = $2\sqrt{2}i$. $14/(2\sqrt{2}i) = 7/(\sqrt{2}i) = -7i/\sqrt{2} = 0 - (7\sqrt{2}/2)i$.

4. Solve $3x^2 - 4x + 20/3 = 0$.

Multiply by 3: $9x^2 - 12x + 20 = 0$. $D = 144 - 720 = -576$. $x = (12 \pm 24i)/18 = 2/3 \pm (4/3)i$.

5. Solve $x^2 + x/\sqrt{2} + 1 = 0$.

Multiply by $\sqrt{2}$: $\sqrt{2}x^2 + x + \sqrt{2} = 0$. $D = 1 - 8 = -7$. $x = (-1 \pm i\sqrt{7})/(2\sqrt{2})$.

6. If $a + ib = (x + i)^2/(2x^2 + 1)$, prove that $a^2 + b^2 = (x^2 + 1)^2/(2x^2 + 1)^2$.

Take modulus squared: $a^2 + b^2 = |x + i|^4/(2x^2 + 1)^2 = (x^2 + 1)^2/(2x^2 + 1)^2$.

D. Long answer question

If $z_1 = 2 - i$ and $z_2 = 1 + i$, find $|(z_1 + z_2 + 1)/(z_1 - z_2 + 1)|$. Also find the real part of $(z_1 z_2)/\bar{z}_1$.

$z_1 + z_2 + 1 = 4$; $z_1 - z_2 + 1 = 2 - 2i$. $|4/(2 - 2i)| = 4/(2\sqrt{2}) = \sqrt{2}$.

$z_1 z_2 = (2 - i)(1 + i) = 2 + 2i - i + 1 = 3 + i$. $\bar{z}_1 = 2 + i$. $(3 + i)/(2 + i) = (3 + i)(2 - i)/5 = (6 - 3i + 2i + 1)/5 = (7 - i)/5$. Real part = $7/5$.

QB-5. Linear Inequalities

Key points

- Adding/subtracting the same number keeps the sign; multiplying/dividing by a negative number reverses it.
- For $x \in \mathbb{N}$ or $\mathbb{Z}$, list only the allowed integers in the answer.
- Combined inequality $a < px + q < b$: work on all three parts at the same time.

A. Multiple choice questions

1. If $-3x + 17 < -13$, then (a) $x \in (10, \infty)$ (b) $x \in [10, \infty)$ (c) $x \in (-\infty, 10]$ (d) $x \in [-10, 10)$

(a) $-3x < -30 \Rightarrow x > 10$.

2. If x is real and $-5x > 20$, then x is (a) > -4 (b) < -4 (c) > 4 (d) < 4 (b).

3. The solution of $|x| \leq 3$ is (a) $[-3, 3]$ (b) $(-3, 3)$ (c) $[0, 3]$ (d) $(-\infty, 3]$ (a).

4. Solution of $2 \leq 3x - 4 \leq 5$ is (a) $[2, 3]$ (b) $(2, 3)$ (c) $[1, 3]$ (d) $[2, 9]$ (a) $6 \leq 3x \leq 9$.

B. Short answer questions

1. Solve $24x < 100$ when (i) $x \in \mathbb{N}$ (ii) $x \in \mathbb{Z}$.

$x < 25/6 \approx 4.17$. (i) $\{1, 2, 3, 4\}$ (ii) $\{\dots, -2, -1, 0, 1, 2, 3, 4\}$.

2. Solve $3(x - 1) \leq 2(x - 3)$.

$3x - 3 \leq 2x - 6 \Rightarrow x \leq -3, (-\infty, -3]$.

3. Solve $x/3 > x/2 + 1$.

Multiply by 6: $2x > 3x + 6 \Rightarrow -x > 6 \Rightarrow x < -6$.

4. Solve $3(2 - x) \geq 2(1 - x)$ and show on the number line.

$6 - 3x \geq 2 - 2x \Rightarrow 4 \geq x \Rightarrow x \leq 4$. Filled circle at 4, shade left.

5. Solve $-12 < 4 - 3x/(-5) \leq 2$, i.e. $-12 < 4 + 3x/5 \leq 2$.

Subtract 4: $-16 < 3x/5 \leq -2$. Multiply by $5/3$: $-80/3 < x \leq -10/3$. Solution $(-80/3, -10/3]$.

6. Solve the system $3x - 7 < 5 + x$, $11 - 5x \leq 1$.

$2x < 12 \Rightarrow x < 6$. $-5x \leq -10 \Rightarrow x \geq 2$. Solution $[2, 6)$.

C. Word problems

1. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution so that the result has more than 4% but less than 6% boric acid. If there are 640 litres of the 8% solution, how many litres of the 2% solution must be added?

Let x litres. $4\% \text{ of } (640 + x) < 8\% \text{ of } 640 + 2\% \text{ of } x < 6\% \text{ of } (640 + x)$.

Left: $0.04(640 + x) < 51.2 + 0.02x \Rightarrow 25.6 + 0.04x < 51.2 + 0.02x \Rightarrow 0.02x < 25.6 \Rightarrow x < 1280$.

Right: $51.2 + 0.02x < 38.4 + 0.06x \Rightarrow 12.8 < 0.04x \Rightarrow x > 320$.

So **more than 320 L and less than 1280 L**.

2. The longest side of a triangle is 3 times the shortest side, and the third side is 2 cm shorter than the longest. If the perimeter is at least 61 cm, find the minimum length of the shortest side.

Shortest x , longest $3x$, third $3x - 2$. $x + 3x + 3x - 2 \geq 61 \Rightarrow 7x \geq 63 \Rightarrow x \geq 9$. Minimum **9 cm**.

3. $IQ = (MA/CA) \times 100$. If $80 \leq IQ \leq 140$ for a group of 12-year-old children, find the range of their mental age.

$80 \leq (MA/12) \times 100 \leq 140 \Rightarrow 9.6 \leq MA \leq 16.8$.

QB-6. Permutations and Combinations

Key points

- $n! = n \times (n - 1)!; 0! = 1$.
- ${}^nP_n = n!; {}^nP_0 = 1; {}^nC_0 = {}^nC_n = 1; {}^nC_1 = n$.
- Things always together $\rightarrow$ treat them as one block, then arrange inside the block.
- Things never together $\rightarrow$ total $-$ (together).
- "At least one" $\rightarrow$ total $-$ none.
- Number of diagonals of an n -sided polygon $= {}^nC_2 - n$. Number of triangles from n points (no three collinear) $= {}^nC_3$.

A. Multiple choice questions

1. $7!/5! =$ (a) 42 (b) 2 (c) 56 (d) 7

(a) 7×6 .

2. ${}^8C_3 =$ (a) 56 (b) 336 (c) 24 (d) 28

(a) $(8 \times 7 \times 6)/6$.

3. Number of ways to arrange the letters of "ROSE" is (a) 4 (b) 12 (c) 24 (d) 16

(c) $4!$.

4. Number of diagonals of a hexagon is (a) 15 (b) 9 (c) 6 (d) 12

(b) $15 - 6$.

5. If ${}^nP_2 = 30$, $n =$ (a) 5 (b) 6 (c) 7 (d) 15

(b) $n(n - 1) = 30$.

6. In how many ways can 3 prizes be given to 5 students if each student can get at most one prize? (a) 10 (b) 60 (c) 125 (d) 15

(b) 5P_3 .

7. Number of triangles formed from 10 points, no three collinear: (a) 120 (b) 720 (c) 30 (d) 45

(a) ${}^{10}C_3$.

8. Number of 3-digit numbers with no digit repeated using 1-9: (a) 729 (b) 504 (c) 84 (d)

27

(b) $9 \times 8 \times 7$.

B. Short answer questions

1. Find n if $(n + 1)! = 12 (n - 1)!$.

$$(n + 1)n = 12 \Rightarrow n^2 + n - 12 = 0 \Rightarrow \mathbf{n = 3}.$$

2. Find r if ${}^5P_r = 2 \cdot {}^6P_{r-1}$.

$$5!/(5 - r)! = 2 \cdot 6!/(7 - r)! \Rightarrow (7 - r)(6 - r) = 12 \Rightarrow r^2 - 13r + 30 = 0 \Rightarrow r = 3 \text{ or } 10. r \leq 5 \Rightarrow \mathbf{r = 3}.$$

3. How many 4-letter words (with or without meaning) can be formed from the letters of "LOGARITHMS" if repetition is not allowed?

$$10 \text{ distinct letters: } {}^{10}P_4 = 5040.$$

4. In how many ways can the letters of "MISSISSIPPI" be arranged? In how many do the four I's not come together?

$$11 \text{ letters: M1, I4, S4, P2. Total} = 11!/(4! 4! 2!) = \mathbf{34650}.$$

$$\text{I's together: treat IIII as one block} \rightarrow 8 \text{ units with S4, P2: } 8!/(4! 2!) = 840. \text{ Not together} = 34650 - 840 = \mathbf{33810}.$$

5. How many words can be formed from the letters of "DAUGHTER" so that (i) vowels are always together (ii) vowels never together?

$$\text{Vowels A, U, E (3); consonants D, G, H, T, R (5). (i) 6 units: } 6! \times 3! = 720 \times 6 = \mathbf{4320}.$$

$$\text{(ii) Total } 8! = 40320; \text{ never together} = 40320 - 4320 = \mathbf{36000}.$$

6. In how many ways can a student choose 5 courses out of 9 if 2 courses are compulsory?

$$\text{Choose 3 from 7: } {}^7C_3 = 35.$$

7. A bag has 5 black and 6 red balls. In how many ways can 2 black and 3 red balls be selected?

$${}^5C_2 \times {}^6C_3 = 10 \times 20 = \mathbf{200}.$$

C. Long answer questions

1. A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways if it has (i) exactly 3 girls (ii) at least 3 girls (iii) at most 3 girls?

$$\text{(i) } {}^4C_3 \times {}^9C_4 = 4 \times 126 = \mathbf{504}.$$

$$\text{(ii) 3 girls: } 504; \text{ 4 girls: } {}^4C_4 \times {}^9C_3 = 84. \text{ Total} = \mathbf{588}.$$

$$\text{(iii) 0 girls: } {}^9C_7 = 36; \text{ 1 girl: } 4 \times {}^9C_6 = 4 \times 84 = 336; \text{ 2 girls: } 6 \times {}^9C_5 = 6 \times 126 = 756; \text{ 3 girls: } 504. \text{ Total} = \mathbf{1632}.$$

2. If the letters of the word "AGAIN" are arranged in all possible ways and listed as in a dictionary, what is the 50th word?

Letters A, A, G, I, N. Words starting with A: $4! = 24$. Starting with G: $4!/2! = 12$ (total 36). Starting with I: 12 (total 48). Then words starting with N: the 49th is NAAGI and the 50th is **NAAIG**.

3. How many numbers greater than 1000000 can be formed using the digits 1, 2, 0, 2, 4, 2, 4?

All 7 digits must be used (7-digit numbers). Total arrangements = $7!/(3! 2!) = 420$ (2 appears 3 times, 4 twice). Those starting with 0: $6!/(3! 2!) = 60$. Required = $420 - 60 = 360$.

4. In how many ways can a cricket team of 11 be selected from 17 players of whom 5 are bowlers, if the team must have exactly 4 bowlers?

${}^5C_4 \times {}^{12}C_7 = 5 \times 792 = 3960$.

QB-7. Binomial Theorem

Key points

- $(a + b)^n$ has $(n + 1)$ terms; powers of a decrease, powers of b increase, sum of powers = n in every term.
- $(a - b)^n$: signs alternate +, -, +, ...
- Coefficients nC_r are symmetric: ${}^nC_r = {}^nC_{n-r}$.
- Sum of all binomial coefficients = 2^n .
- To find a term containing x^k or the term independent of x: write T_{r+1} , collect powers of x, equate to k (or 0).

A. Multiple choice questions

1. The number of terms in $(1 + 2x)^{10}$ is (a) 10 (b) 11 (c) 12 (d) 20
(b).

2. The middle term of $(x + 1/x)^{10}$ is (a) T_5 (b) T_6 (c) T_7 (d) T_5 and T_6
(b) $n/2 + 1 = 6$; its value is ${}^{10}C_5 = 252$.

3. Sum of the coefficients in $(1 + x)^8$ is (a) 8 (b) 64 (c) 256 (d) 128
(c) 2^8 .

4. The 3rd term of $(2 + x)^5$ is (a) $80x^2$ (b) $40x^2$ (c) $10x^2$ (d) $80x^3$
(a) ${}^5C_2 \cdot 2^3 \cdot x^2 = 10 \times 8x^2$.

B. Short answer questions

1. Expand $(x/3 + 1/x)^5$.

$$= x^5/243 + 5(x^4/81)(1/x) + 10(x^3/27)(1/x^2) + 10(x^2/9)(1/x^3) + 5(x/3)(1/x^4) + 1/x^5$$

$$= x^5/243 + 5x^3/81 + 10x/27 + 10/(9x) + 5/(3x^3) + 1/x^5.$$

2. Evaluate $(\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$.

$$(a + b)^6 - (a - b)^6 = 2[6a^5b + 20a^3b^3 + 6ab^5]. \quad a = \sqrt{3}, b = \sqrt{2}: a^5b = 9\sqrt{3} \cdot \sqrt{2} = 9\sqrt{6};$$

$$a^3b^3 = 3\sqrt{3} \cdot 2\sqrt{2} = 6\sqrt{6}; ab^5 = \sqrt{3} \cdot 4\sqrt{2} = 4\sqrt{6}. \text{ Sum} = 2[54\sqrt{6} + 120\sqrt{6} + 24\sqrt{6}] =$$

$$\mathbf{396\sqrt{6}}.$$

3. Evaluate $(102)^5$.

$$(100 + 2)^5 = 10^{10} + 5 \cdot 10^8 \cdot 2 + 10 \cdot 10^6 \cdot 4 + 10 \cdot 10^4 \cdot 8 + 5 \cdot 10^2 \cdot 16 + 32 = 10000000000$$

$$+ 10000000000 + 400000000 + 800000 + 8000 + 32 = \mathbf{11040808032}.$$

4. Show that $9^{n+1} - 8n - 9$ is divisible by 64 for every positive integer n .

$$9^{n+1} = (1 + 8)^{n+1} = 1 + (n + 1)8 + {}^{n+1}C_2 8^2 + \dots \text{ So } 9^{n+1} - 8n - 9 = 64[{}^{n+1}C_2 +$$

$${}^{n+1}C_3 \cdot 8 + \dots], \text{ a multiple of 64.}$$

5. Find the coefficient of x^6y^3 in $(x + 2y)^9$.

$$T_4 = {}^9C_3 x^6 (2y)^3 = 84 \times 8 = \mathbf{672}.$$

6. Find the term independent of x in $(x^2 - 1/x)^9$.

$$T_{r+1} = {}^9C_r x^{18-2r} (-1)^r x^{-r} = {}^9C_r (-1)^r x^{18-3r}. \quad 18 - 3r = 0 \Rightarrow r = 6. \text{ Term} = {}^9C_6 =$$

$$\mathbf{84}.$$

C. Long answer question

The coefficients of three consecutive terms in $(1 + x)^n$ are in the ratio 1 : 3 : 5. Find n .

$$\text{Let them be } {}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}. \quad {}^nC_r / {}^nC_{r-1} = (n - r + 1)/r = 3 \Rightarrow n - 4r + 1 = 0.$$

$${}^nC_{r+1} / {}^nC_r = (n - r)/(r + 1) = 5/3 \Rightarrow 3n - 8r - 5 = 0. \text{ From the first, } n = 4r - 1;$$

$$\text{substitute: } 12r - 3 - 8r - 5 = 0 \Rightarrow r = 2, \mathbf{n = 7}.$$

QB-8. Sequences and Series

Key points

- A.P.: $a, a + d, a + 2d, \dots; a_n = a + (n - 1)d$. Three numbers in A.P.: $a - d, a, a + d$.
- G.P.: $a, ar, ar^2, \dots$; three numbers in G.P.: $a/r, a, ar$.
- If a, b, c are in G.P., $b^2 = ac$. If in A.P., $2b = a + c$.

- $S_{\infty} = a/(1 - r)$ exists only when $-1 < r < 1$.
- Useful sums: $\Sigma n = n(n + 1)/2$; $\Sigma n^2 = n(n + 1)(2n + 1)/6$; $\Sigma n^3 = [n(n + 1)/2]^2$.

A. Multiple choice questions

- 1. The 8th term of the G.P. 0.3, 0.06, 0.012, ... is (a) $0.3(0.2)^7$ (b) $0.3(0.2)^8$ (c) $0.06(0.2)^7$ (d) $0.3(5)^7$**
(a) $r = 0.2$.
- 2. The G.M. of 4 and 9 is (a) 6.5 (b) 6 (c) 36 (d) 13**
(b) $\sqrt{36}$.
- 3. Sum to infinity of $6 + 1.2 + 0.24 + \dots$ is (a) 7.5 (b) 7 (c) 8 (d) 30**
(a) $6/(1 - 0.2)$.
- 4. If 2, x, 8 are in G.P., $x =$ (a) 5 (b) ± 4 (c) 16 (d) 6**
(b) $x^2 = 16$.
- 5. $1 + 2 + 3 + \dots + 50 =$ (a) 1250 (b) 1275 (c) 2550 (d) 1225**
(b) $50 \times 51/2$.

B. Short answer questions

- 1. Find the 20th and n-th terms of the G.P. $5/2, 5/4, 5/8, \dots$**

$$a = 5/2, r = 1/2. a_{20} = (5/2)(1/2)^{19} = 5/2^{20}; a_n = 5/2^n.$$

- 2. The 4th term of a G.P. is the square of its 2nd term, and the first term is -3 . Find the 7th term.**

$$ar^3 = (ar)^2 \Rightarrow r = a = -3. a_7 = -3(-3)^6 = -2187.$$

- 3. Find the sum of 7 terms of $1, -a, a^2, -a^3, \dots$ ($a \neq -1$).**

$$r = -a: S_7 = [1 - (-a)^7]/(1 + a) = (1 + a^7)/(1 + a).$$

- 4. How many terms of the G.P. $3, 3^2, 3^3, \dots$ are needed to give the sum 120?**

$$3(3^n - 1)/2 = 120 \Rightarrow 3^n - 1 = 80 \Rightarrow 3^n = 81 \Rightarrow n = 4.$$

- 5. Evaluate $\Sigma_{k=1}^{11} (2 + 3^k)$.**

$$= 22 + (3 + 3^2 + \dots + 3^{11}) = 22 + 3(3^{11} - 1)/2 = 22 + (3^{12} - 3)/2 = 22 + (531441 - 3)/2 = 22 + 265719 = 265741.$$

- 6. If the A.M. and G.M. of two positive numbers are 10 and 8, find the numbers.**

$$a + b = 20, ab = 64. \text{ Roots of } x^2 - 20x + 64 = 0: x = 16, 4. \text{ Numbers } 16 \text{ and } 4.$$

C. Long answer questions

1. Find the sum to n terms of the sequence 8, 88, 888, 8888, ...

$$S = (8/9)(9 + 99 + 999 + \dots) = (8/9)[(10 - 1) + (100 - 1) + \dots] = (8/9)[(10 + 10^2 + \dots + 10^n) - n] = (8/9)[10(10^n - 1)/9 - n] = \mathbf{(80/81)(10^n - 1) - 8n/9}.$$

2. The sum of first three terms of a G.P. is $39/10$ and their product is 1. Find the common ratio and the terms.

Terms $a/r, a, ar$: product $a^3 = 1 \Rightarrow a = 1$. $1/r + 1 + r = 39/10 \Rightarrow 10r^2 - 29r + 10 = 0 \Rightarrow (5r - 2)(2r - 5) = 0 \Rightarrow r = 2/5$ or $5/2$. Terms: $\mathbf{5/2, 1, 2/5}$ (or in reverse).

3. Find the sum of n terms of the series $1 \times 2 + 2 \times 3 + 3 \times 4 + \dots$

$$a_k = k(k + 1) = k^2 + k. S_n = \Sigma k^2 + \Sigma k = n(n + 1)(2n + 1)/6 + n(n + 1)/2 = n(n + 1)[(2n + 1) + 3]/6 = \mathbf{n(n + 1)(n + 2)/3}.$$

4. The population of a town is 20,000 and increases by 5% every year. Find the population after 3 years.

G.P. with $r = 1.05$: $20000 \times (1.05)^3 = 20000 \times 1.157625 = \mathbf{23,152.5 \approx 23,153}$.

QB-9. Straight Lines

Key points

- Slope of a horizontal line = 0; of a vertical line = not defined.
- Line through origin: $y = mx$. Line parallel to $ax + by + c = 0$: $ax + by + k = 0$.
Perpendicular: $bx - ay + k = 0$.
- Normal form: $x \cos \omega + y \sin \omega = p$ (p = perpendicular distance from origin, ω = angle of the perpendicular with the x-axis).
- Collinearity: area of triangle = 0 or slopes equal.
- Point of intersection: solve the two equations simultaneously.

A. Multiple choice questions

- 1. The slope of the line through (2, 3) and (4, 7) is (a) $1/2$ (b) 2 (c) 4 (d) -2**
(b) $4/2$.
- 2. The inclination of the line $\sqrt{3}x - y + 5 = 0$ is (a) 30° (b) 45° (c) 60° (d) 120°**
(c) slope $\sqrt{3} = \tan 60^\circ$.
- 3. The lines $2x + 3y = 5$ and $3x - 2y = 7$ are (a) parallel (b) perpendicular (c) coincident (d) none**
(b) slopes $-2/3$ and $3/2$, product -1 .
- 4. The y-intercept of $3x + 4y = 12$ is (a) 4 (b) 3 (c) 12 (d) -3**
(b) put $x = 0$.
- 5. Distance of (0, 0) from $3x + 4y - 10 = 0$ is (a) 10 (b) 2 (c) 5 (d) 1**
(b) $10/5$.
- 6. If the points (1, 2), (3, 4) and (k, 7) are collinear, $k =$ (a) 6 (b) 5 (c) 7 (d) 4**
(a) slope $1 \Rightarrow (7 - 4)/(k - 3) = 1$.
- 7. Equation of the x-axis is (a) $x = 0$ (b) $y = 0$ (c) $x + y = 0$ (d) $x = y$**
(b).

B. Short answer questions

1. Find the equation of the line through $(-4, 3)$ with slope $1/2$.

$$y - 3 = \frac{1}{2}(x + 4) \Rightarrow 2y - 6 = x + 4 \Rightarrow \mathbf{x - 2y + 10 = 0}.$$

2. Find the equation of the line that cuts off intercepts a and b such that $a + b = 9$ and $ab = 20$ ($a > b$).

$$a = 5, b = 4: x/5 + y/4 = 1 \Rightarrow \mathbf{4x + 5y = 20}.$$

3. Reduce $\sqrt{3}x + y - 8 = 0$ to normal form and find p and ω .

$$\text{Divide by } \sqrt{(3 + 1)} = 2: (\sqrt{3}/2)x + (1/2)y = 4 \Rightarrow x \cos 30^\circ + y \sin 30^\circ = 4. \quad p = 4, \omega = 30^\circ.$$

4. Find the angle between $x - \sqrt{3}y = 5$ and $\sqrt{3}x + y = 1$.

Slopes $1/\sqrt{3}$ and $-\sqrt{3}$; product $= -1 \Rightarrow 90^\circ$.

5. Find the point of intersection of $3x + y = 9$ and $2x - y = 1$.

Adding: $5x = 10 \Rightarrow x = 2, y = 3$. **(2, 3)**.

6. Find k so that the line through $(3, k)$ and $(2, 7)$ is parallel to the line through $(-1, 4)$ and $(0, 6)$.

Slope of second $= 2$. $(7 - k)/(2 - 3) = 2 \Rightarrow 7 - k = -2 \Rightarrow k = 9$.

7. Find the distance between $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

$|-34 - 31|/\sqrt{(225 + 64)} = 65/17$.

8. Find the equation of the perpendicular bisector of the segment joining $(3, 4)$ and $(-1, 2)$.

Mid-point $(1, 3)$. Slope of segment $= 2/4 = 1/2 \Rightarrow$ perpendicular slope -2 . $y - 3 = -2(x - 1) \Rightarrow 2x + y - 5 = 0$.

C. Long answer questions

1. Find the equations of the lines through $(3, 2)$ making an angle of 45° with the line $x - 2y = 3$.

Slope of given line $m_1 = 1/2$. $\tan 45^\circ = |(m - 1/2)/(1 + m/2)| = 1 \Rightarrow (2m - 1)/(2 + m) = \pm 1$.

Case +1: $2m - 1 = 2 + m \Rightarrow m = 3 \Rightarrow y - 2 = 3(x - 3) \Rightarrow 3x - y - 7 = 0$.

Case -1: $2m - 1 = -2 - m \Rightarrow m = -1/3 \Rightarrow y - 2 = -(x - 3)/3 \Rightarrow x + 3y - 9 = 0$.

2. Find the foot of the perpendicular from $(-1, 3)$ to the line $3x - 4y - 16 = 0$.

Perpendicular through $(-1, 3)$: $4x + 3y + k = 0 \Rightarrow -4 + 9 + k = 0 \Rightarrow k = -5$: $4x + 3y - 5 = 0$. Solve with $3x - 4y = 16$: multiply: $12x + 9y = 15$ and $12x - 16y = 64 \Rightarrow 25y = -49 \Rightarrow y = -49/25$; $x = (5 - 3y)/4 = (5 + 147/25)/4 = (272/25)/4 = 68/25$. Foot = **$(68/25, -49/25)$** .

3. The vertices of a triangle are $A(2, 3)$, $B(4, -1)$, $C(1, 2)$. Find the equation and length of the altitude from A .

Slope $BC = (2 + 1)/(1 - 4) = -1 \Rightarrow$ altitude slope 1 : $y - 3 = x - 2 \Rightarrow x - y + 1 = 0$.

Line BC : $y + 1 = -(x - 4) \Rightarrow x + y - 3 = 0$. Length $= |2 + 3 - 3|/\sqrt{2} = 2/\sqrt{2} = \sqrt{2}$ units.

4. Prove that the product of the perpendiculars from $(\sqrt{a^2 - b^2}, 0)$ and $(-\sqrt{a^2 - b^2}, 0)$ to the line $(x/a)\cos \theta + (y/b)\sin \theta = 1$ is b^2 .

Write the line as $bx \cos \theta + ay \sin \theta - ab = 0$. Let $c = \sqrt{a^2 - b^2}$. $p_1 = |bc \cos \theta - ab| / \sqrt{(b^2 \cos^2 \theta + a^2 \sin^2 \theta)}$, $p_2 = |-bc \cos \theta - ab| / \sqrt{(\dots)}$. $p_1 p_2 = b^2 |a^2 - c^2 \cos^2 \theta| / (b^2 \cos^2 \theta + a^2 \sin^2 \theta)$. Now $a^2 - (a^2 - b^2) \cos^2 \theta = a^2 \sin^2 \theta + b^2 \cos^2 \theta$, which equals the denominator. Hence $p_1 p_2 = b^2$.

QB-10. Conic Sections

Key points

Curve	Standard equation	Key relation	e	Latus rectum
Circle	$(x - h)^2 + (y - k)^2 = r^2$	—	0	—
Parabola	$y^2 = 4ax$	focus $(a, 0)$, directrix $x = -a$	1	$4a$
Ellipse	$x^2/a^2 + y^2/b^2 = 1$	$c^2 = a^2 - b^2$	$c/a < 1$	$2b^2/a$
Hyperbola	$x^2/a^2 - y^2/b^2 = 1$	$c^2 = a^2 + b^2$	$c/a > 1$	$2b^2/a$

A. Multiple choice questions

- Centre of $x^2 + y^2 + 8x + 10y - 8 = 0$ is (a) (4, 5) (b) (-4, -5) (c) (8, 10) (d) (-8, -10)
(b) $(-g, -f)$.
- Radius of $x^2 + y^2 - 6x = 0$ is (a) 6 (b) 3 (c) 9 (d) $\sqrt{6}$
(b) $\sqrt{9}$.
- Focus of $x^2 = -16y$ is (a) (0, 4) (b) (0, -4) (c) (4, 0) (d) (-4, 0)
(b) $a = 4$, opens downward.
- Length of the latus rectum of $y^2 = 10x$ is (a) 10 (b) 5 (c) 2.5 (d) 40
(a) $4a$.
- Eccentricity of $x^2/16 - y^2/9 = 1$ is (a) $5/4$ (b) $4/5$ (c) $3/4$ (d) $5/3$
(a) $c = 5$, $a = 4$.
- Length of minor axis of $x^2/49 + y^2/36 = 1$ is (a) 14 (b) 12 (c) 6 (d) 7
(b) $2b$.
- The conic with $e = 1$ is (a) circle (b) ellipse (c) parabola (d) hyperbola
(c).

B. Short answer questions

1. Find the equation of the circle with centre $(1/2, 1/4)$ and radius $1/12$.

$(x - 1/2)^2 + (y - 1/4)^2 = 1/144 \Rightarrow x^2 + y^2 - x - y/2 + 1/4 + 1/16 - 1/144 = 0 \Rightarrow$
multiply by 144: **$144x^2 + 144y^2 - 144x - 72y + 44 = 0$** , i.e. $36x^2 + 36y^2 - 36x - 18y + 11 = 0$.

2. Find the circle passing through (0, 0), (a, 0), (0, b).

$x^2 + y^2 + 2gx + 2fy + c = 0$. $(0,0) \Rightarrow c = 0$. $(a, 0) \Rightarrow a^2 + 2ga = 0 \Rightarrow g = -a/2$. $(0, b) \Rightarrow f = -b/2$. Circle: $x^2 + y^2 - ax - by = 0$.

3. Does the point (-2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?

$6.25 + 12.25 = 18.5 < 25 \Rightarrow$ inside.

4. Find the equation of the parabola with vertex (0, 0), passing through (2, 3), axis along the x-axis.

$y^2 = 4ax \Rightarrow 9 = 8a \Rightarrow a = 9/8$. $y^2 = 9x/2$, i.e. $2y^2 = 9x$.

5. Find the equation of the ellipse with foci $(\pm 3, 0)$ and $a = 4$.

$b^2 = 16 - 9 = 7$: $x^2/16 + y^2/7 = 1$.

6. Find the equation of the ellipse with major axis on the y-axis, passing through (3, 2) and (1, 6).

$x^2/b^2 + y^2/a^2 = 1$. Let $p = 1/b^2$, $q = 1/a^2$: $9p + 4q = 1$, $p + 36q = 1$. From the second $p = 1 - 36q$: $9 - 324q + 4q = 1 \Rightarrow q = 1/40$, $p = 1/10$. $x^2/10 + y^2/40 = 1$.

7. Find the hyperbola with vertices $(\pm 2, 0)$ and foci $(\pm 3, 0)$.

$b^2 = 9 - 4 = 5$: $x^2/4 - y^2/5 = 1$.

C. Long answer questions**1. Find the coordinates of the foci and vertices, the eccentricity and the length of the latus rectum of $36x^2 + 4y^2 = 144$.**

Divide by 144: $x^2/4 + y^2/36 = 1$. Major axis along y: $a = 6$, $b = 2$, $c = \sqrt{36 - 4} = 4\sqrt{2}$. Foci $(0, \pm 4\sqrt{2})$; vertices $(0, \pm 6)$; $e = 4\sqrt{2}/6 = 2\sqrt{2}/3$; latus rectum $= 2(4)/6 = 4/3$.

2. Find the equation of the hyperbola with foci $(0, \pm\sqrt{10})$ passing through (2, 3).

$y^2/a^2 - x^2/b^2 = 1$, $a^2 + b^2 = 10$. $9/a^2 - 4/b^2 = 1$. Put $b^2 = 10 - a^2$: $9(10 - a^2) - 4a^2 = a^2(10 - a^2) \Rightarrow 90 - 13a^2 = 10a^2 - a^4 \Rightarrow a^4 - 23a^2 + 90 = 0 \Rightarrow a^2 = 5$ or 18 . $a^2 = 18$ makes b^2 negative, so $a^2 = 5$, $b^2 = 5$. $y^2/5 - x^2/5 = 1$.

3. An arch is in the form of a parabola with its axis vertical. The arch is 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex?

Take vertex at origin, opening downward: $x^2 = -4ay$. Base end points $(\pm 5/2, -10)$: $25/4 = 40a \Rightarrow a = 5/32$. At $y = -2$: $x^2 = 4(5/32)(2) = 5/4 \Rightarrow x = \pm\sqrt{5}/2$. Width $= \sqrt{5} \approx 2.24$ m.

QB-11. Three-Dimensional Geometry

A. Multiple choice questions

1. The point $(4, -2, -5)$ lies in octant (a) IV (b) V (c) VIII (d) VII

(c) $(+, -, -)$.

2. Distance of $(3, 4, 12)$ from the origin is (a) 13 (b) 19 (c) 12 (d) 5

(a) $\sqrt{(9 + 16 + 144)} = \sqrt{169}$.

3. A point on the y-axis has coordinates (a) $(x, 0, 0)$ (b) $(0, y, 0)$ (c) $(0, 0, z)$ (d) $(x, y, 0)$
(b).

4. The equation of the XY-plane is (a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) $x + y = 0$
(c).

B. Short answer questions

1. Find the distance between $(2, -1, 3)$ and $(-2, 1, 3)$.

$$\sqrt{(16 + 4 + 0)} = 2\sqrt{5}.$$

2. Show that $(0, 7, -10)$, $(1, 6, -6)$ and $(4, 9, -6)$ are vertices of an isosceles triangle.

$$AB = \sqrt{(1 + 1 + 16)} = \sqrt{18}; BC = \sqrt{(9 + 9 + 0)} = \sqrt{18}; AC = \sqrt{(16 + 4 + 16)} = 6. AB = BC \Rightarrow \text{isosceles}.$$

3. Find the equation of the set of points P such that $PA^2 + PB^2 = 2k^2$, where $A = (3, 4, 5)$, $B = (-1, 3, -7)$.

$$(x - 3)^2 + (y - 4)^2 + (z - 5)^2 + (x + 1)^2 + (y - 3)^2 + (z + 7)^2 = 2k^2 \Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = 2k^2.$$

4. Verify that $(-1, 2, 1)$, $(1, -2, 5)$, $(4, -7, 8)$ and $(2, -3, 4)$ are vertices of a parallelogram.

Mid-point of AC = $(3/2, -5/2, 9/2)$; mid-point of BD = $(3/2, -5/2, 9/2)$. Diagonals bisect each other $\Rightarrow$ parallelogram.

QB-12. Limits and Derivatives

Key points

- Always try direct substitution first.
- $0/0$ form: factorise, rationalise, or use standard limits ($\sin x/x \rightarrow 1$, $(x^n - a^n)/(x - a) \rightarrow na^{n-1}$).
- For limits at infinity of rational functions, divide numerator and denominator by the highest power of x .
- Derivative from first principles: $f'(x) = \lim_{h \rightarrow 0} [f(x + h) - f(x)]/h$.
- $d/dx(\sqrt{x}) = 1/(2\sqrt{x})$; $d/dx(1/x) = -1/x^2$.

A. Multiple choice questions

1. $\lim_{x \rightarrow 2} (x^2 - 4)/(x - 2) =$ (a) 0 (b) 2 (c) 4 (d) not defined

(c).

2. $\lim_{x \rightarrow 0} \sin 3x/\sin 5x =$ (a) 1 (b) $3/5$ (c) $5/3$ (d) 0

(b).

3. $\lim_{x \rightarrow 0} \tan x/x =$ (a) 0 (b) 1 (c) ∞ (d) -1

(b).

4. $\lim_{x \rightarrow 1} (x^5 - 1)/(x - 1) =$ (a) 1 (b) 4 (c) 5 (d) 0

(c) $5 \cdot 1^4$.

5. Derivative of x^5 at $x = 1$ is (a) 1 (b) 5 (c) 4 (d) 0

(b).

6. $d/dx(\sin x \cos x) =$ (a) $\cos 2x$ (b) $\sin 2x$ (c) $-\sin 2x$ (d) $\cos^2 x$

(a) $\frac{1}{2} \sin 2x$ differentiates to $\cos 2x$.

7. $\lim_{x \rightarrow \infty} (3x^2 + 1)/(x^2 - 5) =$ (a) 0 (b) 3 (c) ∞ (d) 1

(b).

8. $d/dx(1/x^2) =$ (a) $-2/x^3$ (b) $2/x^3$ (c) $-1/x$ (d) $1/(2x)$

(a).

B. Very short answer questions

1. Evaluate $\lim_{x \rightarrow \pi} (x - 22/7)$.

$\pi - 22/7$.

2. Evaluate $\lim_{x \rightarrow 0} \cos x/(\pi - x)$.

$1/\pi$.

3. Find the derivative of $99x$ at $x = 100$.

99.

4. Differentiate $(x - 1)(x - 2)$.

$x^2 - 3x + 2 \Rightarrow 2x - 3$.

C. Short answer questions

1. Evaluate $\lim_{x \rightarrow 3} (x^4 - 81)/(2x^2 - 5x - 3)$.

$$(x^2 - 9)(x^2 + 9)/[(2x + 1)(x - 3)] = (x + 3)(x^2 + 9)/(2x + 1) \rightarrow 6 \times 18/7 = \mathbf{108/7}.$$

2. Evaluate $\lim_{x \rightarrow 0} [(x + 1)^5 - 1]/x$.

$$\text{Put } y = x + 1 \rightarrow 1: (y^5 - 1)/(y - 1) \rightarrow 5.$$

3. Evaluate $\lim_{x \rightarrow 0} (\cos 2x - 1)/(\cos x - 1)$.

$$-2\sin^2 x / (-2\sin^2(x/2)) \rightarrow [x/(x/2)]^2 = 4.$$

4. Evaluate $\lim_{x \rightarrow 0} x \sec x$.

$$0 \times 1 = 0.$$

5. Evaluate $\lim_{x \rightarrow 0} (\sin ax + bx)/(ax + \sin bx)$.

$$\text{Divide by } x: (a \cdot (\sin ax/ax) + b)/(a + b \cdot (\sin bx/bx)) \rightarrow (a + b)/(a + b) = 1.$$

6. Find $f'(x)$ from first principles for $f(x) = 3x + 2$.

$$[3(x + h) + 2 - 3x - 2]/h = 3.$$

7. Differentiate from first principles $f(x) = \cos x$.

$$[\cos(x + h) - \cos x]/h = -2 \sin(x + h/2) \sin(h/2)/h = -\sin(x + h/2) \cdot [\sin(h/2)/(h/2)] \rightarrow -\sin x.$$

8. Differentiate $(x^2 + 1)/x$.

$$x + 1/x \Rightarrow 1 - 1/x^2.$$

9. Differentiate $(ax + b)/(cx + d)$.

$$[a(cx + d) - c(ax + b)]/(cx + d)^2 = (ad - bc)/(cx + d)^2.$$

10. Differentiate $\sin^n x$ (use product rule idea for $n = 2, 3$).

$$\sin^2 x = \sin x \cdot \sin x \Rightarrow 2 \sin x \cos x = \sin 2x. \text{ In general } d/dx \sin^n x = n \sin^{n-1} x \cos x.$$

D. Long answer questions

1. If $f(x) = |x| + 1$ ($x < 0$), 0 ($x = 0$), $|x| - 1$ ($x > 0$), for what value of a does $\lim_{x \rightarrow a} f(x)$ exist?

For $x < 0$: $f(x) = -x + 1$; for $x > 0$: $f(x) = x - 1$. At $a = 0$: LHL = 1, RHL = -1, so the limit does not exist at 0. For any $a \neq 0$ the function is a polynomial near a , so the limit exists. Hence the limit exists for all $\mathbf{a \neq 0}$.

2. Find $\lim_{x \rightarrow 1} f(x)$ if $f(x) = x^2 - 1$ ($x \leq 1$), $-x^2 - 1$ ($x > 1$).

LHL = $1 - 1 = 0$; RHL = $-1 - 1 = -2$. Unequal = **limit does not exist**.

3. Find the derivative of $(x + \sec x)(x - \tan x)$.

Product rule: $(1 + \sec x \tan x)(x - \tan x) + (x + \sec x)(1 - \sec^2 x)$.

4. Find the derivative of $(\sec x - 1)/(\sec x + 1)$.

Write as $(1 - \cos x)/(1 + \cos x)$ (multiply top and bottom by $\cos x$). Derivative = $[\sin x(1 + \cos x) + \sin x(1 - \cos x)]/(1 + \cos x)^2 = \mathbf{2 \sin x/(1 + \cos x)^2}$.

5. A particle moves so that its distance $s = t^3 - 6t^2 + 9t$ metres after t seconds. Find its velocity at $t = 2$ s and when it is momentarily at rest.

$v = ds/dt = 3t^2 - 12t + 9$. At $t = 2$: $12 - 24 + 9 = \mathbf{-3 \text{ m/s}}$. At rest when $v = 0$: $3(t - 1)(t - 3) = 0 \Rightarrow \mathbf{t = 1 \text{ s and } t = 3 \text{ s}}$.

QB-13. Statistics

Key points

- Mean deviation is least when taken about the median.
- Variance and standard deviation do not change if a constant is added to each observation; if each is multiplied by k , σ is multiplied by $|k|$ and variance by k^2 .
- Variance of first n natural numbers = $(n^2 - 1)/12$.
- Lower C.V. $\Rightarrow$ more consistent / less variable.

A. Multiple choice questions

1. Range of 12, 7, 19, 4, 15 is (a) 15 (b) 19 (c) 11 (d) 8

(a) $19 - 4$.

2. If $\sigma = 4$, the variance is (a) 2 (b) 8 (c) 16 (d) 4

(c).

3. If each observation is multiplied by 3, the S.D. becomes (a) same (b) 3 times (c) 9 times (d) $1/3$

(b).

4. Mean = 50, S.D. = 10. C.V. = (a) 5% (b) 20% (c) 50% (d) 10%

(b) $10/50 \times 100$.

5. Variance of 2, 4, 6, 8, 10 is (a) 8 (b) 6 (c) 4 (d) 10

(a) mean 6; deviations squared 16, 4, 0, 4, 16 $\rightarrow 40/5$.

B. Short answer questions

1. Find the mean deviation about the median of 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17.

Sorted: 10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18. $n = 12$, median = $(13 + 14)/2 = 13.5$. $|x - 13.5|$: 3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5 $\rightarrow$ sum 28. M.D. = $28/12 = 2.33$.

2. Find the mean deviation about the mean for the data below.

x	5	10	15	20	25
f	7	4	6	3	5

$N = 25$. $\Sigma fx = 35 + 40 + 90 + 60 + 125 = 350 \Rightarrow \bar{x} = 14$. $|x - 14|$: 9, 4, 1, 6, 11. $\Sigma f|x - 14| = 63 + 16 + 6 + 18 + 55 = 158$. M.D. = $158/25 = 6.32$.

3. The mean of 5 observations is 4.4 and the variance is 8.24. If three are 1, 2, 6, find the other two.

Sum = 22 $\Rightarrow x + y = 13$. $\Sigma x^2 = 5(8.24 + 19.36) = 138 \Rightarrow x^2 + y^2 = 138 - 41 = 97$. $xy = (169 - 97)/2 = 36 \Rightarrow 9 \text{ and } 4$.

4. Find the variance of the first 10 multiples of 3.

3, 6, ..., 30 = $3 \times (1, \dots, 10)$. Variance of 1..10 = $99/12 = 8.25$; multiply by 9: **74.25**.

C. Long answer questions

1. Calculate the mean, variance and S.D. using the step-deviation method.

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
f	3	7	12	15	8	3	2

Mid-points x : 35, 45, 55, 65, 75, 85, 95. Take $A = 65$, $h = 10$: $y = -3, -2, -1, 0, 1, 2, 3$. $N = 50$.

$\Sigma fy = -9 - 14 - 12 + 0 + 8 + 6 + 6 = -15$. $\Sigma fy^2 = 27 + 28 + 12 + 0 + 8 + 12 + 18 = 105$.

Mean = $65 + 10(-15/50) = 65 - 3 = 62$.

Variance = $100[105/50 - (-0.3)^2] = 100[2.1 - 0.09] = 201$; $\sigma = \sqrt{201} \approx 14.18$.

2. The following are the marks of two students A and B in 5 tests. Who is more consistent?

A	58	59	60	65	66
B	56	87	89	46	93

A: mean = $308/5 = 61.6$; deviations $-3.6, -2.6, -1.6, 3.4, 4.4$; squares 12.96, 6.76, 2.56, 11.56, 19.36 $\rightarrow 53.2$; $\sigma_A = \sqrt{10.64} = 3.26$; C.V. = 5.3%.

B: mean = $371/5 = 74.2$; deviations $-18.2, 12.8, 14.8, -28.2, 18.8$; squares 331.24, 163.84, 219.04, 795.24, 353.44 $\rightarrow 1862.8$; $\sigma_B = \sqrt{372.56} = 19.30$; C.V. = 26.0%.

A is more consistent (lower C.V.).

QB-14. Probability

Key points

- $0 \leq P(E) \leq 1$; $P(S) = 1$; $P(\varphi) = 0$.
- Tossing n coins: 2^n outcomes. Throwing two dice: 36 outcomes. A pack has 52 cards: 4 suits of 13; 12 face cards (J, Q, K); 4 aces.
- Odds in favour of $E = P(E) : P(\text{not } E)$.
- $P(A \cup B \cup C) = \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$.

A. Multiple choice questions

1. Probability of getting a sum 7 with two dice is (a) 1/6 (b) 7/36 (c) 5/36 (d) 1/12

(a) 6 outcomes out of 36.

2. Probability of drawing a face card from a pack is (a) 3/13 (b) 1/13 (c) 4/13 (d) 1/4

(a) 12/52.

3. Three coins are tossed. P(exactly two heads) = (a) 1/8 (b) 3/8 (c) 1/2 (d) 1/4

(b) HHT, HTH, THH.

4. If $P(E) = 0.05$, $P(\text{not } E) =$ (a) 0.05 (b) 0.5 (c) 0.95 (d) 0.9

(c).

5. A leap year is chosen at random. Probability that it has 53 Sundays is (a) 1/7 (b) 2/7 (c) 3/7 (d) 53/366

(b) 2 extra days: 7 possible pairs, 2 contain Sunday.

6. If A and B are mutually exclusive with $P(A) = 0.3$ and $P(B) = 0.4$, $P(A \cup B) =$ (a) 0.12 (b) 0.7 (c) 0.58 (d) 0.1

(b).

B. Short answer questions

1. Describe the sample space for tossing a coin and then throwing a die.

$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$, 12 outcomes.

2. A die is rolled. $E = \{1, 3, 5\}$, $F = \{2, 3\}$, $G = \{2, 3, 4, 5\}$. Find $E \cap F$, $E \cup G$, and check whether E and F are mutually exclusive.

$E \cap F = \{3\}$; $E \cup G = \{1, 2, 3, 4, 5\}$. Since $E \cap F \neq \varnothing$, E and F are not mutually exclusive.

3. A letter is chosen at random from "ASSASSINATION". Find the probability that it is a vowel.

13 letters; vowels A, A, I, A, I, O = 6. $P = \frac{6}{13}$.

4. Given $P(A) = \frac{3}{5}$, $P(B) = \frac{1}{5}$, A and B mutually exclusive. Find $P(A \text{ or } B)$.

$\frac{4}{5}$.

5. In a lottery, a person chooses 6 different numbers from 1 to 20. If these match the 6 numbers fixed by the committee, he wins. Find the probability of winning.

$\frac{1}{{}^{20}C_6} = \frac{1}{38760}$.

C. Long answer questions

1. In a class, 40% students study Mathematics, 25% Biology and 15% both. One student is selected at random. Find the probability that the student studies (i) Mathematics or Biology (ii) neither (iii) only Mathematics.

(i) $0.40 + 0.25 - 0.15 = \frac{5}{10}$. (ii) $1 - 0.5 = \frac{5}{10}$. (iii) $0.40 - 0.15 = \frac{25}{100}$.

2. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100, what is the probability that (i) you both enter the same section (ii) you enter different sections?

Total ways to choose 40 for section A: ${}^{100}C_{40}$. Both in A: ${}^{98}C_{38}$; both in B: ${}^{98}C_{40}$.

$P(\text{same}) = \frac{{}^{98}C_{38} + {}^{98}C_{40}}{{}^{100}C_{40}} = \frac{(40 \times 39 + 60 \times 59)}{(100 \times 99)} = \frac{(1560 + 3540)}{9900} = \frac{17}{33}$.

$P(\text{different}) = 1 - \frac{17}{33} = \frac{16}{33}$.

3. A bag contains 9 discs: 4 red, 3 blue, 2 yellow. One disc is drawn. Find $P(\text{red})$, $P(\text{yellow})$, $P(\text{blue})$, $P(\text{not blue})$, $P(\text{red or blue})$.

$\frac{4}{9}$, $\frac{2}{9}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{7}{9}$.

SAMPLE PAPERS WITH SOLUTIONS

Mathematics · +2 1st Year · Full marks 80 · Time 3 hours

HOW TO PRACTISE

Sit at a table with a 3-hour timer, attempt the whole paper without looking at the solutions, then check and give yourself marks step by step, as an examiner would.

Sample Paper 1

Section A (1 mark each)

1. Write the set of all vowels in the word "EQUATION".

{E, U, A, I, O}.

2. If $A = \{1, 2\}$ and $B = \{a\}$, find $B \times A$.

$\{(a, 1), (a, 2)\}$.

3. Convert 25° into radians.

$25\pi/180 = 5\pi/36$.

4. Find the value of i^{-35} .

$1/i^{35} = 1/i^3 = 1/(-i) = i$.

5. Find the value of ${}^{12}C_{10}$.

${}^{12}C_2 = 66$.

6. Find the 5th term of the G.P. 2, 6, 18, ...

$2 \times 3^4 = 162$.

7. Find the slope of the line joining (3, -2) and (7, -2).

0.

8. Find the equation of the circle with centre (0, 2) and radius 2.

$x^2 + (y - 2)^2 = 4$, i.e. $x^2 + y^2 - 4y = 0$.

9. Evaluate $\lim_{x \rightarrow 0} \sin 2x/x$.

2.

10. Differentiate $x^{-4}(3 - 4x^{-5})$.

$$y = 3x^{-4} - 4x^{-9} \Rightarrow y' = -12x^{-5} + 36x^{-10}.$$

11. Find the range of 8, 3, 15, 9, 1, 12.

$$15 - 1 = 14.$$

12. A coin is tossed twice. Find the probability of getting no head.

$$1/4.$$

Section B (2 marks each)

13. If $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$ and $B = \{3, 4, 5\}$, find $A' \cap B'$ and verify it equals $(A \cup B)'$.

$$A' = \{1, 4, 5, 6\}, B' = \{1, 2, 6\}; A' \cap B' = \{1, 6\}. A \cup B = \{2, 3, 4, 5\}; (A \cup B)' = \{1, 6\}. \text{ Equal.}$$

14. Find the domain of $f(x) = 1/\sqrt{x - 2}$.

$$x - 2 > 0 \Rightarrow (2, \infty).$$

15. Find the value of $\cos 15^\circ$.

$$\cos(45^\circ - 30^\circ) = (1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(1/2) = (\sqrt{3} + 1)/(2\sqrt{2}).$$

16. Solve $5x - 3 < 3x + 1$ for $x \in \mathbb{Z}$.

$$2x < 4 \Rightarrow x < 2: \{\dots, -1, 0, 1\}.$$

17. How many 3-digit even numbers can be formed from 1, 2, 3, 4, 6, 7 if no digit is repeated?

$$\text{Units digit: 2, 4 or 6 (3 ways); then } 5 \times 4 \text{ for the other places. } 3 \times 5 \times 4 = 60.$$

18. Find the distance of $(-1, 1)$ from the line $12(x + 6) = 5(y - 2)$.

$$\text{Line: } 12x - 5y + 82 = 0. d = |-12 - 5 + 82|/13 = 65/13 = 5.$$

19. Find the derivative of $x \sin x$.

$$\sin x + x \cos x.$$

20. A card is drawn from a well-shuffled pack. Find the probability that it is neither a heart nor a king.

$$P(\text{heart or king}) = 16/52 = 4/13. \text{ Required} = 9/13.$$

Section C (4 marks each)

21. Prove that $(\sin x + \sin 3x)/(\cos x + \cos 3x) = \tan 2x$.

$$\text{Numerator } 2 \sin 2x \cos x; \text{ denominator } 2 \cos 2x \cos x. \text{ Ratio } \tan 2x.$$

22. Find the modulus and the real and imaginary parts of $(1 + 2i)/(1 - 3i)$.

$(1 + 2i)(1 + 3i)/10 = (1 + 3i + 2i - 6)/10 = (-5 + 5i)/10 = -1/2 + i/2$. Real $-1/2$, imaginary $1/2$, modulus $\sqrt{(1/4 + 1/4)} = 1/\sqrt{2}$.

23. Find the 13th term in the expansion of $(9x - 1/(3\sqrt{x}))^{18}$.

$$T_{13} = {}^{18}C_{12}(9x)^6(-1/(3\sqrt{x}))^{12} = 18564 \times 9^6 x^6 \times 1/(3^{12} x^6) = 18564 \times 3^{12}/3^{12} = \mathbf{18564}.$$

24. Find the sum of 20 terms of the series $5 + 55 + 555 + \dots$

$$S = (5/9)[(10 + 100 + \dots + 10^{20}) - 20] = (5/9)[10(10^{20} - 1)/9 - 20] = (50/81)(10^{20} - 1) - 100/9.$$

25. Find the equation of the line through the intersection of $2x + y = 5$ and $x + 3y = -8$ and parallel to $3x + 4y = 7$.

Solve: from the first $y = 5 - 2x$; $x + 15 - 6x = -8 \Rightarrow x = 23/5$, $y = 5 - 46/5 = -21/5$.

Parallel line $3x + 4y = k$: $69/5 - 84/5 = -3 = k$. $\mathbf{3x + 4y + 3 = 0}$.

26. Find the derivative of $\cos x$ from first principles.

See QB-12, short answer 7: $-\sin x$.

Section D (6 marks each)**27. Prove: $\cos 4x = 1 - 8 \cos^2 x + 8 \cos^4 x$. OR Prove: $\sin 3x = 3 \sin x - 4 \sin^3 x$.**

$$\cos 4x = 2\cos^2 2x - 1 = 2(2\cos^2 x - 1)^2 - 1 = 2(4\cos^4 x - 4\cos^2 x + 1) - 1 = 8\cos^4 x - 8\cos^2 x + 1.$$

$$\text{OR: } \sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x = 2 \sin x \cos^2 x + (1 - 2\sin^2 x)\sin x \\ x = 2 \sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x = 3 \sin x - 4 \sin^3 x.$$

28. Find the foci, vertices, length of major and minor axes, eccentricity and latus rectum of $16x^2 + y^2 = 16$.

$x^2/1 + y^2/16 = 1$. $a = 4$ (along y), $b = 1$, $c = \sqrt{15}$. Foci $(0, \pm\sqrt{15})$, vertices $(0, \pm 4)$, major axis 8, minor axis 2, $e = \sqrt{15}/4$, latus rectum $= 2(1)/4 = 1/2$.

29. Find the mean and variance of the following data.

Class	0-10	10-20	20-30	30-40	40-50
f	5	8	15	16	6

x : 5, 15, 25, 35, 45; $A = 25$, $h = 10$; $y = -2, -1, 0, 1, 2$. $N = 50$. $\Sigma fy = -10 - 8 + 0 + 16 + 12 = 10$. $\Sigma fy^2 = 20 + 8 + 0 + 16 + 24 = 68$.

Mean $= 25 + 10(10/50) = \mathbf{27}$. Variance $= 100[68/50 - (0.2)^2] = 100(1.36 - 0.04) = \mathbf{132}$.

Sample Paper 2

Section A (1 mark each)

1. Is the set of points on a line finite or infinite?

Infinite.

2. Find the range of $f(x) = 3 \sin x$.

$[-3, 3]$.

3. Find the conjugate of $(2 - i)^2$.

$(2 - i)^2 = 3 - 4i$; conjugate $3 + 4i$.

4. If ${}^nC_{12} = {}^nC_8$, find n .

20.

5. Find the number of terms in $(2x - 3y)^{12}$.

13.

6. Find the sum to infinity of $1 - 1/2 + 1/4 - 1/8 + \dots$

$1/(1 + 1/2) = 2/3$.

7. Write the equation of the line parallel to the y-axis through $(-3, 4)$.

$x = -3$.

8. Find the length of the latus rectum of $x^2 = 12y$.

12.

9. Name the octant of $(-4, 2, -5)$.

VI.

10. Evaluate $\lim_{x \rightarrow 1} (x^2 + 1)/(x + 100)$.

$2/101$.

11. If the variance is 25, find the S.D.

5.

12. If $P(A) = 2/11$, find the odds in favour of A.

$2 : 9$.

Section B (2 marks each)

13. List all subsets of $\{-1, 0, 1\}$.

$\varnothing, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{-1, 1\}, \{0, 1\}, \{-1, 0, 1\}$ — 8 subsets.

14. If $f(x) = x^2$ and $g(x) = 2x + 1$, find $(f - g)(2)$ and $(f/g)(1)$.

$(f - g)(2) = 4 - 5 = -1$. $(f/g)(1) = 1/3$.

15. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm.

$\theta = 22/100 \text{ rad} = 0.22 \times 180/\pi = 0.22 \times 180 \times 7/22 = 12.6^\circ = 12^\circ 36'$.

16. Solve $x^2 + 3x + 9 = 0$.

$$D = 9 - 36 = -27. x = (-3 \pm 3\sqrt{3} i)/2.$$

17. Find the number of chords that can be drawn through 21 points on a circle.

$${}^{21}C_2 = 210.$$

18. Find the equation of the line with slope $-1/3$ and y-intercept -4 .

$$y = -x/3 - 4 \Rightarrow x + 3y + 12 = 0.$$

19. Evaluate $\lim_{x \rightarrow 0} (1 - \cos x)/x^2$.

$$2\sin^2(x/2)/x^2 = (1/2)[\sin(x/2)/(x/2)]^2 \rightarrow 1/2.$$

20. Find the probability of getting a doublet when two dice are thrown.

$$6/36 = 1/6.$$

Section C (4 marks each)

21. In a group of 70 people, 37 like coffee, 52 like tea and each likes at least one. How many like both? How many like coffee only?

$$70 = 37 + 52 - x \Rightarrow x = 19. \text{ Coffee only} = 37 - 19 = 18.$$

22. Prove: $(\cos x - \cos y)/(\sin x + \sin y) = -\tan((x - y)/2)$.

Numerator $-2 \sin((x + y)/2) \sin((x - y)/2)$; denominator $2 \sin((x + y)/2) \cos((x - y)/2)$.
Ratio $-\tan((x - y)/2)$.

23. Solve $2 \leq 3x - 4 \leq 5$ and $-3 \leq 4 - 7x/2 \leq 18$.

First: $6 \leq 3x \leq 9 \Rightarrow [2, 3]$. Second: $-7 \leq -7x/2 \leq 14 \Rightarrow$ multiply by $-2/7$ (reverse): $2 \geq x \geq -4 \Rightarrow [-4, 2]$.

24. The sum of the first 6 terms of a G.P. is 9 times the sum of its first 3 terms. Find the common ratio.

$$a(r^6 - 1)/(r - 1) = 9a(r^3 - 1)/(r - 1) \Rightarrow r^3 + 1 = 9 \Rightarrow r^3 = 8 \Rightarrow \mathbf{r = 2}.$$

25. Find the equation of the parabola with focus (6, 0) and directrix $x = -6$, and the coordinates of the ends of its latus rectum.

$$y^2 = 24x. \text{ Latus rectum ends } (6, \pm 12).$$

26. Differentiate $(x^2 \cos(\pi/4))/\sin x$.

$$(1/\sqrt{2}) \cdot [2x \sin x - x^2 \cos x] / \sin^2 x.$$

Section D (6 marks each)

27. In how many ways can the letters of "PERMUTATIONS" be arranged if (i) words start with P and end with S (ii) vowels are all together (iii) there are always 4 letters between P and S?

12 letters, T twice.

(i) Middle 10 letters with T twice: $10!/2! = 1814400$.

(ii) 2419200 (worked earlier in the notes).

(iii) Positions for (P, S) with 4 letters between: P at 1-7 gives 7 positions; P and S can swap: 14. Remaining 10 letters: $10!/2! = 1814400$. Total = $14 \times 1814400 = 25401600$.

28. Find the equation of the circle passing through (2, -2) and (3, 4) with centre on $x + y = 2$.

Centre (h, k), $h + k = 2$. $(2 - h)^2 + (-2 - k)^2 = (3 - h)^2 + (4 - k)^2 \Rightarrow 8 - 4h + 4k = 25 - 6h - 8k \Rightarrow 2h + 12k = 17$. With $h = 2 - k$: $4 - 2k + 12k = 17 \Rightarrow k = 1.3$, $h = 0.7$. $r^2 = (1.3)^2 + (3.3)^2 = 1.69 + 10.89 = 12.58$. Circle: $(x - 0.7)^2 + (y - 1.3)^2 = 12.58$.

29. A committee of two persons is selected from two men and two women. Find the probability that the committee has (i) no man (ii) one man (iii) two men.

Total ${}^4C_2 = 6$. (i) $1/6$ (ii) $4/6 = 2/3$ (iii) $1/6$.

Practice Exercises with Answers

Try these on your own. Answers are given at the end of each set.

Set 1 - Sets, Relations, Functions

- Write $\{x : x \in \mathbb{N}, x \text{ is a divisor of } 12\}$ in roster form.
- If $A = \{x : x \text{ is a prime less than } 10\}$, find $n(P(A))$.
- If $n(U) = 50$, $n(A) = 20$, $n(B) = 25$, $n(A \cap B) = 8$, find $n(A' \cap B')$.
- Find the domain of $f(x) = \sqrt{x^2 - 9}$.
- If $f(x) = (x - 1)/(x + 1)$, find $f(f(x))$.
- Find the range of $f(x) = 2 - 3x$, $x \in \mathbb{R}$, $x > 0$.
- If $A = \{1, 2, 3\}$, how many relations are there on A?
- Find $[2.999] + [-2.001]$.

Answers: 1. $\{1, 2, 3, 4, 6, 12\}$ 2. 16 3. 13 4. $(-\infty, -3] \cup [3, \infty)$ 5. $-1/x$ 6. $(-\infty, 2)$ 7. $2^9 = 512$ 8. $2 + (-3) = -1$.

Set 2 - Trigonometry

- Convert $-47^\circ 30'$ into radians.
- Find $\sin(-11\pi/3)$.
- If $\tan x = -4/3$, x in quadrant II, find $\sin x$ and $\cos x$.
- Prove $\sin(n + 1)x \sin(n + 2)x + \cos(n + 1)x \cos(n + 2)x = \cos x$.

5. Find the value of $\sin 105^\circ$.
6. Prove $(1 + \cos 2x)/\sin 2x = \cot x$.
7. Find the maximum value of $5 \sin x - 12 \cos x$.
8. Prove $\sin 2x + 2 \sin 4x + \sin 6x = 4 \cos^2 x \sin 4x$.

Answers: 1. $-19\pi/72$ 2. $\sqrt{3}/2$ 3. $\sin x = 4/5$, $\cos x = -3/5$ 4. Use $\cos(A - B)$ 5. $(\sqrt{3} + 1)/(2\sqrt{2})$ 6. $2\cos^2 x/(2 \sin x \cos x)$ 7. 13 8. $\sin 2x + \sin 6x = 2 \sin 4x \cos 2x$; total $2 \sin 4x(\cos 2x + 1) = 4 \sin 4x \cos^2 x$.

Set 3 - Algebra

1. Express $(-2 - i/3)^3$ in $a + ib$ form.
2. Find the multiplicative inverse of $-i$.
3. Solve $3x^2 - 4x + 20/3 = 0$.
4. Solve $(2x - 1)/3 \geq (3x - 2)/4 - (2 - x)/5$.
5. Find n if ${}^{2n}C_3 : {}^nC_3 = 12 : 1$.
6. How many words, with or without meaning, can be made from the letters of "EQUATION" using each letter once?
7. Find the coefficient of x^5 in $(1 + 2x)^6$.
8. Find the sum of 10 terms of the G.P. 1, 3, 9, ...
9. Insert two numbers between 3 and 81 so that the sequence is a G.P.
10. Find the 10th term of the G.P. $\sqrt{2}$, 2, $2\sqrt{2}$, ...

Answers: 1. $-22/3 - (107/27)i$ 2. i 3. $2/3 \pm 4i/3$ 4. $x \leq 2$, i.e. $(-\infty, 2]$ 5. $n = 5$ 6. $8! = 40320$ 7. ${}^6C_5 \cdot 2^5 = 192$ 8. $(3^{10} - 1)/2 = 29524$ 9. 9, 27 10. $(\sqrt{2})^{10} = 32$.

Set 4 - Coordinate Geometry, Calculus, Statistics, Probability

1. Find the equation of the line through (1, 2) and (3, 6).
2. Find the angle between $y = \sqrt{3}x + 5$ and $y = x/\sqrt{3} - 2$.
3. Find the centre and radius of $2x^2 + 2y^2 - x = 0$.
4. Find the eccentricity of $4x^2 + 9y^2 = 36$.
5. Find the distance between (1, 2, 3) and (4, 6, 15).
6. Evaluate $\lim_{x \rightarrow 0} (\sqrt{1+x} - \sqrt{1-x})/x$.
7. Differentiate $(x+1)(x+2)(x+3)$.
8. Find the mean deviation about the mean of 4, 7, 8, 9, 10, 12, 13, 17.
9. Two dice are thrown. Find $P(\text{sum is a prime number})$.
10. Find $P(\text{at least one head})$ when 3 coins are tossed.

Answers: 1. $2x - y = 0$ 2. 30° 3. $(1/4, 0)$, $r = 1/4$ 4. $\sqrt{5}/3$ 5. 13 6. 1 7. $3x^2 + 12x + 11$ 8. mean 10; $\Sigma|x - 10| = 24$; M.D. = 3 9. sums 2, 3, 5, 7, 11: $1 + 2 + 4 + 6 + 2 = 15 \Rightarrow 15/36 = 5/12$ 10. $7/8$.

MORE SOLVED EXAMPLES

Mathematics · +2 1st Year · Textbook-level and higher-order problems

Sets and Functions – Additional Examples

1. Show that for any sets A and B, $A = (A \cap B) \cup (A - B)$.

$$(A \cap B) \cup (A \cap B') = A \cap (B \cup B') = A \cap U = A.$$

2. Show that $A \cup (B - A) = A \cup B$.

$$A \cup (B \cap A') = (A \cup B) \cap (A \cup A') = (A \cup B) \cap U = A \cup B.$$

3. Is it true that for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$?

No. Take $A = \{1\}$, $B = \{2\}$. $P(A) \cup P(B) = \{\emptyset, \{1\}, \{2\}\}$, but $P(A \cup B)$ also contains $\{1, 2\}$. So they are not equal in general.

4. Let $f = \{(x, x^2/(1 + x^2)) : x \in \mathbb{R}\}$. Find the range of f.

$$y = x^2/(1 + x^2) \Rightarrow y + yx^2 = x^2 \Rightarrow x^2 = y/(1 - y). \text{ For real } x \text{ we need } y/(1 - y) \geq 0 \Rightarrow 0 \leq y < 1. \text{ Range} = [0, 1).$$

5. Let $f(x) = x^2$ and $g(x) = 2x + 1$ be functions from $\mathbb{R}$ to $\mathbb{R}$. Find $(f + g)$, $(f - g)$, (fg) , (f/g) .

$$(f + g)(x) = x^2 + 2x + 1; (f - g)(x) = x^2 - 2x - 1; (fg)(x) = 2x^3 + x^2; (f/g)(x) = x^2/(2x + 1), x \neq -1/2.$$

6. The function $f(x) = 9x/5 + 32$ converts Celsius to Fahrenheit. Find $f(0)$, $f(-10)$, and the value of x for which $f(x) = 212$.

$$f(0) = 32; f(-10) = -18 + 32 = 14; 9x/5 + 32 = 212 \Rightarrow 9x/5 = 180 \Rightarrow x = 100.$$

7. Find the domain and range of $f(x) = \sqrt{9 - x^2}$.

$9 - x^2 \geq 0 \Rightarrow -3 \leq x \leq 3$: domain $[-3, 3]$. The value $\sqrt{9 - x^2}$ varies from 0 (at $x = \pm 3$) to 3 (at $x = 0$): range $[0, 3]$.

Trigonometry – Additional Examples**1. The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes? ($\pi = 3.14$)**

In 60 min it turns 2π ; in 40 min, $4\pi/3$ rad. $l = r\theta = 1.5 \times 4\pi/3 = 2\pi = \mathbf{6.28 \text{ cm}}$.

2. If arcs of the same length in two circles subtend angles 60° and 75° at the centres, find the ratio of their radii.

$r_1(\pi/3) = r_2(5\pi/12) \Rightarrow r_1/r_2 = (5/12)/(1/3) = 5/4$. Ratio **5 : 4**.

3. Prove: $\cos(\pi/4 - x) \cos(\pi/4 - y) - \sin(\pi/4 - x) \sin(\pi/4 - y) = \sin(x + y)$.

LHS = $\cos[(\pi/4 - x) + (\pi/4 - y)] = \cos(\pi/2 - (x + y)) = \sin(x + y)$.

4. Prove: $\tan(\pi/4 + x)/\tan(\pi/4 - x) = [(1 + \tan x)/(1 - \tan x)]^2$.

$\tan(\pi/4 + x) = (1 + \tan x)/(1 - \tan x)$; $\tan(\pi/4 - x) = (1 - \tan x)/(1 + \tan x)$. Their ratio = $[(1 + \tan x)/(1 - \tan x)]^2$.

5. Prove: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4 \cos x \cos 2x \sin 4x$.

$(\sin 7x + \sin x) + (\sin 5x + \sin 3x) = 2 \sin 4x \cos 3x + 2 \sin 4x \cos x = 2 \sin 4x (\cos 3x + \cos x) = 2 \sin 4x \times 2 \cos 2x \cos x$.

6. Prove: $(\sin 5x - 2 \sin 3x + \sin x)/(\cos 5x - \cos x) = \tan x$.

Numerator: $(\sin 5x + \sin x) - 2 \sin 3x = 2 \sin 3x \cos 2x - 2 \sin 3x = 2 \sin 3x (\cos 2x - 1) = -4 \sin 3x \sin^2 x$. Denominator: $-2 \sin 3x \sin 2x = -4 \sin 3x \sin x \cos x$. Ratio = $\sin x/\cos x = \tan x$.

7. Find the value of $\tan(\pi/8)$.

$\tan 2\theta = 2t/(1 - t^2)$ with $2\theta = \pi/4$: $1 = 2t/(1 - t^2) \Rightarrow t^2 + 2t - 1 = 0 \Rightarrow t = -1 \pm \sqrt{2}$. $\pi/8$ is in quadrant I, so $\tan(\pi/8) = \mathbf{\sqrt{2} - 1}$.

8. If $\tan x = 3/4$ and $\pi < x < 3\pi/2$, find $\sin 2x$, $\cos 2x$, $\tan 2x$.

$\sin x = -3/5$, $\cos x = -4/5$. $\sin 2x = 2(-3/5)(-4/5) = 24/25$; $\cos 2x = 16/25 - 9/25 = 7/25$; $\tan 2x = 24/7$.

Complex Numbers, P & C, Binomial – Additional Examples

1. Find the real numbers x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

$(x - iy)(3 + 5i) = 3x + 5y + i(5x - 3y) = -6 + 24i$. So $3x + 5y = -6$ and $5x - 3y = 24$.
Solving: multiply first by 3 and second by 5: $9x + 15y = -18$, $25x - 15y = 120 \Rightarrow 34x = 102 \Rightarrow x = 3$, $y = -3$.

2. If $(a + ib)/(c + id) = x + iy$, prove that $(a^2 + b^2)/(c^2 + d^2) = x^2 + y^2$.

Take modulus on both sides and square: $|a + ib|^2/|c + id|^2 = |x + iy|^2$.

3. Find the least positive integer m such that $((1 + i)/(1 - i))^m = 1$.

$(1 + i)/(1 - i) = i$. $i^m = 1 \Rightarrow$ least $m = 4$.

4. How many words with or without meaning, each of 3 vowels and 2 consonants, can be formed from the letters of "DAUGHTER"?

Choose 3 vowels from 3 (A, U, E): 1 way; 2 consonants from 5: 10 ways; arrange 5 letters: $5! = 120$. Total = 1200.

5. In how many ways can one select a cricket team of 11 from 17 players, of whom only 5 can bowl, if each team must include exactly 4 bowlers?

${}^5C_4 \times {}^{12}C_7 = 5 \times 792 = 3960$.

6. Find the number of arrangements of the letters of "INDEPENDENCE" in which (i) words start with P (ii) all vowels are together (iii) vowels are never together (iv) words begin with I and end in P.

Letters: I1, N3, D2, E4, P1, C1 (12).

(i) Remaining 11: $11!/(3! 2! 4!) = 138600$.

(ii) Vowels I, E, E, E, E as a block: $5!/4! = 5$ arrangements inside; outside 8 units (block, N3, D2, P, C): $8!/(3! 2!) = 3360$. Total = 16800.

(iii) Total $12!/(3! 2! 4!) = 1663200$; never together = $1663200 - 16800 = 1646400$.

(iv) Middle 10 letters: N3, D2, E4, C1: $10!/(3! 2! 4!) = 12600$.

7. Using binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25.

$6^n = (1 + 5)^n = 1 + 5n + {}^nC_2 5^2 + \dots \Rightarrow 6^n - 5n = 1 + 25k$ for some integer k . Hence the remainder is 1.

8. Find a if the 17th and 18th terms of $(2 + a)^{50}$ are equal.

$T_{17} = {}^{50}C_{16} 2^{34} a^{16}$, $T_{18} = {}^{50}C_{17} 2^{33} a^{17}$. Equal $\Rightarrow a = 2^{50} C_{16} / {}^{50}C_{17} = 2 \times 17/34 = 1$.

Sequences, Coordinate Geometry – Additional Examples

1. If a, b, c, d are in G.P., prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

Let $b = ar, c = ar^2, d = ar^3$. The three expressions become $a^n(1 + r^n), a^n r^n(1 + r^n), a^n r^{2n}(1 + r^n)$ — each is r^n times the previous, so they form a G.P.

2. Find the sum of the products of corresponding terms of 2, 4, 8, 16, 32 and 128, 32, 8, 2, $1/2$.

$$256 + 128 + 64 + 32 + 16 = 496.$$

3. A person has 2 parents, 4 grandparents, 8 great-grandparents and so on. Find the number of his ancestors in the 10 generations preceding his own.

$$2 + 4 + \dots (10 \text{ terms}) = 2(2^{10} - 1) = 2046.$$

4. Find the area of the triangle formed by the lines $y - x = 0, x + y = 0$ and $x - k = 0$.

Vertices $(0, 0), (k, k), (k, -k)$. Area = $\frac{1}{2}|0 + k(-k - 0) + k(0 - k)| = \frac{1}{2}|-2k^2| = k^2$.

5. Find the values of k for which the line $(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$ is (a) parallel to the x-axis (b) parallel to the y-axis (c) passing through the origin.

(a) coefficient of $x = 0 \Rightarrow k = 3$. (b) coefficient of $y = 0 \Rightarrow k = \pm 2$. (c) constant = 0 $\Rightarrow k^2 - 7k + 6 = 0 \Rightarrow k = 1$ or 6 .

6. A rod of length 12 cm moves with its ends always touching the coordinate axes. Find the equation of the locus of a point P on the rod which is 3 cm from the end in contact with the x-axis.

Let the rod meet the x-axis at A and y-axis at B, making angle θ with the x-axis. P is 3 cm from A and 9 cm from B, so $x = 9 \cos \theta$ and $y = 3 \sin \theta$. Hence $x^2/81 + y^2/9 = 1$ — an ellipse.

7. Find the equation of the set of points P, the sum of whose distances from A(4, 0, 0) and B(-4, 0, 0) is 10.

$\sqrt{(x - 4)^2 + y^2 + z^2} + \sqrt{(x + 4)^2 + y^2 + z^2} = 10$. Squaring step by step gives $9x^2 + 25y^2 + 25z^2 = 225$.

Calculus, Statistics, Probability – Additional Examples

1. Evaluate $\lim_{x \rightarrow 0} (\sin(2 + x) - \sin(2 - x))/x$.

Numerator = $2 \cos 2 \sin x$. So the limit = $2 \cos 2 \times 1 = 2 \cos 2$.

2. Evaluate $\lim_{x \rightarrow a} (\sqrt{a+2x} - \sqrt{3x})/(\sqrt{3a+x} - 2\sqrt{x})$.

Rationalise both. Numerator $\rightarrow (a+2x-3x)/(\sqrt{a+2x} + \sqrt{3x}) = (a-x)/(\dots)$.

Denominator $\rightarrow (3a+x-4x)/(\sqrt{3a+x} + 2\sqrt{x}) = 3(a-x)/(\dots)$. Ratio = $(1/3)(\sqrt{3a+x} + 2\sqrt{x})/(\sqrt{a+2x} + \sqrt{3x}) \rightarrow (1/3)(4\sqrt{a})/(2\sqrt{3a}) = \mathbf{2/(3\sqrt{3})}$.

3. Find the derivative of $f(x) = x^n$ from first principles (n a positive integer).

$[(x+h)^n - x^n]/h = [nx^{n-1}h + {}^nC_2x^{n-2}h^2 + \dots]/h = nx^{n-1} + (\text{terms containing } h) \rightarrow nx^{n-1}$.

4. For a group of 200 candidates, the mean and S.D. of scores were 40 and 15. Later it was found that 43 was misread as 34 and 35 as 53. Find the correct mean and S.D.

Wrong $\Sigma x = 8000$. Correct $\Sigma x = 8000 - 34 - 53 + 43 + 35 = 7991 \Rightarrow \text{mean} = 39.955$.

Wrong $\Sigma x^2 = 200(225 + 1600) = 365000$. Correct $\Sigma x^2 = 365000 - 1156 - 2809 + 1849 + 1225 = 364109$.

Variance = $364109/200 - (39.955)^2 = 1820.545 - 1596.402 = 224.143$; S.D. $\approx \mathbf{14.97}$.

5. A die has 1 on two faces, 2 on three faces and 3 on one face. It is rolled once. Find $P(2)$, $P(1 \text{ or } 3)$, $P(\text{not } 3)$.

$P(2) = 3/6 = 1/2$; $P(1 \text{ or } 3) = 3/6 = 1/2$; $P(\text{not } 3) = 5/6$.

6. If 4-digit numbers greater than 5000 are formed randomly from 0, 1, 3, 5, 7, find the probability of forming a number divisible by 5 when (i) repetition is allowed (ii) not allowed.

(i) First digit 5 or 7: $2 \times 5 \times 5 \times 5 = 250$, minus 5000 itself = 249 numbers. Divisible by 5 (last digit 0 or 5): $2 \times 5 \times 5 \times 2 = 100$, minus 5000 = 99. $P = 99/249 = 33/83$.

(ii) First digit 5 or 7 (2 ways), others from remaining 4 digits: $2 \times 4 \times 3 \times 2 = 48$.

Divisible by 5: first 5, last 0: $3 \times 2 = 6$; first 7, last 0 or 5: $2 \times 3 \times 2 = 12$; total 18. $P = 18/48 = 3/8$.

Sample Paper 3

Section A (1 mark each)

1. Write the interval $(-5, 3]$ in set-builder form.

$$\{x : x \in \mathbb{R}, -5 < x \leq 3\}.$$

2. If $n(A) = 5$, find the number of elements of $A \times A$.
25.

3. Find the value of $\tan 225^\circ$.

$$\tan(180^\circ + 45^\circ) = 1.$$

4. Find the additive inverse of $3 - 2i$.

$$-3 + 2i.$$

5. Evaluate $0! + 1!$.

$$2.$$

6. Write the general term of $(x + y)^n$.

$$T_{r+1} = {}^nC_r x^{n-r} y^r.$$

7. Find the G.M. of $1/4$ and 16 .

$$\sqrt{4} = 2.$$

8. Write the slope-intercept form of $2x + 3y = 6$.

$$y = -2x/3 + 2.$$

9. Find the vertex of the parabola $y^2 = -8x$.

$$(0, 0).$$

10. Evaluate $\lim_{x \rightarrow 0} (e^x - 1)/x$.

$$1.$$

11. If each value of a data set is increased by 10, what happens to its S.D.?

It does not change.

12. What is the probability of an impossible event?

$$0.$$

Section B (2 marks each)

13. If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, find $A - B$ and $B - A$.

$$A - B = \{3, 6, 9, 15, 18, 21\}; B - A = \{4, 8, 16, 20\}.$$

14. Let $f(x) = x + 1$ and $g(x) = 2x - 3$. Find $(f \cdot g)(x)$ and $(f/g)(x)$.

$$(f \cdot g)(x) = 2x^2 - x - 3; (f/g)(x) = (x + 1)/(2x - 3), x \neq 3/2.$$

15. Prove: $\sin^2(\pi/6) + \cos^2(\pi/3) - \tan^2(\pi/4) = -1/2$.

$$1/4 + 1/4 - 1 = -1/2.$$

16. Find the modulus of $(1 - i)^{10}$.

$$(\sqrt{2})^{10} = 32.$$

17. In how many ways can 5 people be seated in a row of 5 chairs?

$$5! = 120.$$

18. Find the equation of the line through $(0, 0)$ and perpendicular to $x - 2y + 3 = 0$.

Slope of given = $1/2 \Rightarrow$ required -2 : $y = -2x$, i.e. $2x + y = 0$.

19. Find the derivative of $\sin x/x$.

$$(x \cos x - \sin x)/x^2.$$

20. Three coins are tossed. Find $P(\text{at most one head})$.

$$TTT, HTT, THT, TTH = 4/8 = 1/2.$$

Section C (4 marks each)

21. If A and B are sets with $n(A) = 3$ and $n(B) = 6$, find the minimum and maximum number of elements in $A \cup B$.

Minimum 6 (when $A \subset B$); maximum 9 (when $A \cap B = \varnothing$).

22. Prove: $\cos 2x \cos(x/2) - \cos 3x \cos(9x/2) = \sin 5x \sin(5x/2)$.

See QB-3, Long answer 1.

23. Solve: $x/(2x + 1) \geq 1/4$.

$x/(2x + 1) - 1/4 \geq 0 \Rightarrow (4x - 2x - 1)/(4(2x + 1)) \geq 0 \Rightarrow (2x - 1)/(2x + 1) \geq 0$. Critical points $-1/2, 1/2$. Sign is positive for $x < -1/2$ or $x \geq 1/2$. Solution: $(-\infty, -1/2) \cup [1/2, \infty)$.

24. Find the middle terms in the expansion of $(3 - x^3/6)^7$.

$$T_4 = -105x^9/8 \text{ and } T_5 = 35x^{12}/48 \text{ (worked in the chapter).}$$

25. Find the equation of the hyperbola with foci $(\pm 5, 0)$ and transverse axis of length 8.

$$c = 5, a = 4, b^2 = 9: x^2/16 - y^2/9 = 1.$$

26. Evaluate $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$.

$$(1 - \cos x)/\sin x = 2\sin^2(x/2)/(2 \sin(x/2) \cos(x/2)) = \tan(x/2) \rightarrow 0.$$

Section D (6 marks each)

27. In a survey of 25 students, 15 had taken Mathematics, 12 Physics and 11 Chemistry; 5 took Maths and Chemistry, 9 Maths and Physics, 4 Physics and Chemistry, and 3 all three. Find the number who took (i) only Chemistry (ii) only Mathematics (iii) only Physics (iv) Physics and Chemistry but not Mathematics (v) none.

(i) $11 - 5 - 4 + 3 = 5$ (ii) $15 - 5 - 9 + 3 = 4$ (iii) $12 - 9 - 4 + 3 = 2$ (iv) $4 - 3 = 1$ (v) At least one = $15 + 12 + 11 - 5 - 9 - 4 + 3 = 23 \Rightarrow$ none = 2.

28. Find the sum to n terms of $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \dots$

$$a_k = (2k + 1)k^2 = 2k^3 + k^2. S_n = 2[n(n + 1)/2]^2 + n(n + 1)(2n + 1)/6 = n^2(n + 1)^2/2 + n(n + 1)(2n + 1)/6 = [n(n + 1)/6][3n(n + 1) + 2n + 1] = \mathbf{n(n + 1)(3n^2 + 5n + 1)/6}.$$

29. The mean and S.D. of 20 observations are 10 and 2. On checking, the observation 8 was found to be wrong. Find the correct mean and S.D. if (i) the wrong item is omitted (ii) it is replaced by 12.

$$\Sigma x = 200; \Sigma x^2 = 20(4 + 100) = 2080.$$

(i) $n = 19, \Sigma x = 192, \Sigma x^2 = 2080 - 64 = 2016$. Mean = $192/19 = 10.105$. Variance = $2016/19 - (10.105)^2 = 106.105 - 102.111 = 3.994 \Rightarrow$ S.D. $\approx \mathbf{2.0}$.

(ii) $\Sigma x = 204, \Sigma x^2 = 2016 + 144 = 2160$. Mean = 10.2. Variance = $108 - 104.04 = 3.96 \Rightarrow$ S.D. = $\mathbf{1.99}$.

Sample Paper 4**Section A (1 mark each)**

1. How many subsets does the empty set have?

One (ϕ itself).

2. Find the domain of $f(x) = |x|$.

$\mathbb{R}$.

3. Find $\sin(765^\circ)$.

$$\sin 45^\circ = 1/\sqrt{2}.$$

4. Find the value of $\sqrt{-16} + 3\sqrt{-25}$.

$$4i + 15i = 19i.$$

5. If ${}^nP_4 = 360$, find n .

$$6 (6 \times 5 \times 4 \times 3).$$

6. Find the 10th term of the A.P. 2, 7, 12, ...

$$2 + 45 = 47.$$

7. Find the equation of the line passing through (2, 2) and (-2, -2).

$$y = x.$$

8. Find the length of the major axis of $x^2/25 + y^2/100 = 1$.

$$2 \times 10 = 20.$$

9. Find the coordinates of the foot of the perpendicular from (3, -4, 5) on the z-axis.
(0, 0, 5).

10. Differentiate $\cos 2x$.

$$-2 \sin 2x.$$

11. Mean of 2, 4, 6, 8, 10 is...

6.

12. If $A \subset B$, $P(A) = 0.3$ and $P(B) = 0.5$, find $P(A \cap B)$.

0.3.

Section B (2 marks each)

13. Write all proper subsets of $\{a, b\}$.

$\varnothing, \{a\}, \{b\}$.

14. Find the range of $f(x) = x^2 + 2$.

$[2, \infty)$.

15. Find the value of $\sin 75^\circ + \cos 75^\circ$.

$$\sqrt{2} \sin(75^\circ + 45^\circ) = \sqrt{2} \sin 120^\circ = \sqrt{2} \times \sqrt{3}/2 = \sqrt{6}/2.$$

16. Express $(1 - i)^{-1}$ in $a + ib$ form.

$$(1 + i)/2 = 1/2 + i/2.$$

17. How many 5-digit telephone numbers can be made with digits 0-9 if each number starts with 67 and no digit repeats?

$$\text{Remaining 3 places from 8 digits: } 8 \times 7 \times 6 = 336.$$

18. Find the centre and radius of $x^2 + y^2 - 8x + 10y - 12 = 0$.

$$\text{Centre } (4, -5); r = \sqrt{(16 + 25 + 12)} = \sqrt{53}.$$

19. Evaluate $\lim_{x \rightarrow -2} (1/x + 1/2)/(x + 2)$.

$$(2 + x)/(2x(x + 2)) = 1/(2x) \rightarrow -1/4.$$

20. Find the probability that a number selected from 1 to 30 is a prime.

$$\text{Primes } \leq 30: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 = 10. P = 1/3.$$

Section C (4 marks each)

21. Let $A = \{1, 2, 3, \dots, 14\}$. Define $R = \{(x, y) : 3x - y = 0, x, y \in A\}$. Write R , its domain, co-domain and range.

$$y = 3x \leq 14 \Rightarrow x = 1, 2, 3, 4. R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}. \text{Domain } \{1, 2, 3, 4\}; \text{co-domain } A; \text{range } \{3, 6, 9, 12\}.$$

22. If $\sin x = 3/5$, $\cos y = -12/13$, x and y both in quadrant II, find $\sin(x + y)$.

$$\cos x = -4/5; \sin y = 5/13. \sin(x + y) = (3/5)(-12/13) + (-4/5)(5/13) = -36/65 - 20/65 = -56/65.$$

23. Convert into polar-type form and find the modulus and argument of $z = -1 - i\sqrt{3}$ (argument in $(-\pi, \pi]$).

$$|z| = 2. \text{ The point } (-1, -\sqrt{3}) \text{ is in quadrant III; reference angle } \pi/3. \arg z = -(\pi - \pi/3) = -2\pi/3. z = 2[\cos(-2\pi/3) + i \sin(-2\pi/3)].$$

24. If the first and n -th terms of a G.P. are a and b and P is the product of n terms, prove that $P^2 = (ab)^n$.

$$P = a \cdot ar \cdot ar^2 \dots ar^{n-1} = a^n r^{n(n-1)/2}. P^2 = a^{2n} r^{n(n-1)} = [a \cdot ar^{n-1}]^n = (ab)^n.$$

25. Find the equation of the line through $(-3, 5)$ perpendicular to the line through $(2, 5)$ and $(-3, 6)$.

$$\text{Slope of given} = 1/(-5) = -1/5 \Rightarrow \text{required slope } 5. y - 5 = 5(x + 3) \Rightarrow 5x - y + 20 = 0.$$

26. Differentiate $(4x + 5 \sin x)/(3x + 7 \cos x)$.

$$[(4 + 5 \cos x)(3x + 7 \cos x) - (4x + 5 \sin x)(3 - 7 \sin x)]/(3x + 7 \cos x)^2.$$

Section D (6 marks each)

27. Prove: $\cos(\pi/4 + x) + \cos(\pi/4 - x) = \sqrt{2} \cos x$, and hence find the value of $\cos 75^\circ + \cos 15^\circ$.

$$\text{LHS} = 2 \cos(\pi/4) \cos x = \sqrt{2} \cos x. \text{ With } x = 30^\circ: \cos 75^\circ + \cos 15^\circ = \sqrt{2} \cos 30^\circ = \sqrt{6}/2.$$

28. A bag contains 5 red, 6 white and 7 black balls. Two balls are drawn at random. Find the probability that (i) both are red (ii) one red and one white (iii) neither is black.

$$\text{Total } {}^{18}C_2 = 153. \text{ (i) } {}^5C_2/153 = 10/153 \text{ (ii) } 5 \times 6/153 = 30/153 = 10/51 \text{ (iii) } {}^{11}C_2/153 = 55/153.$$

29. Find the coordinates of the foci and vertices, the eccentricity and the length of the latus rectum of $5y^2 - 9x^2 = 36$.

$$y^2/(36/5) - x^2/4 = 1. a^2 = 36/5, b^2 = 4, c^2 = 56/5 \Rightarrow c = 2\sqrt{14}/\sqrt{5}. \text{ Foci } (0, \pm 2\sqrt{14}/\sqrt{5}); \text{ vertices } (0, \pm 6/\sqrt{5}); e = c/a = (2\sqrt{14}/\sqrt{5})/(6/\sqrt{5}) = \sqrt{14}/3; \text{ latus rectum} = 2b^2/a = 8/(6/\sqrt{5}) = 4\sqrt{5}/3.$$

OBJECTIVE QUESTION BANK

Mathematics · +2 1st Year · 120 MCQs with answers and reasons

TIP

In objective questions, first eliminate clearly wrong options, then check the remaining ones by substituting simple values ($x = 0, 1, \pi/4, \dots$). Answers are given right after each block.

Block 1 – Sets, Relations, Functions (Q1–20)

- Which is a well-defined set? (a) tall students (b) rich people (c) vowels of English (d) good books
- $\{x : x \in \mathbb{Z}, x^2 = 1\} =$ (a) $\{1\}$ (b) $\{-1, 1\}$ (c) $\{0, 1\}$ (d) $\varnothing$
- Number of subsets of a set with 5 elements: (a) 10 (b) 25 (c) 32 (d) 31
- $A \cap \varnothing =$ (a) A (b) $\varnothing$ (c) U (d) A'
- $A \cup U =$ (a) A (b) U (c) $\varnothing$ (d) A'
- If $A = \{1, 3, 5\}$, $B = \{2, 4\}$, $A \cap B =$ (a) $\{1, 2\}$ (b) $\varnothing$ (c) $\{5\}$ (d) A
- $(A \cup B)' =$ (a) $A' \cup B'$ (b) $A' \cap B'$ (c) $A \cap B$ (d) $A - B$
- $U - A =$ (a) A (b) A' (c) $\varnothing$ (d) U
- Interval for $\{x : 0 \leq x < 4\}$ is (a) $(0, 4)$ (b) $[0, 4]$ (c) $[0, 4)$ (d) $(0, 4]$
- $n(A) = 10$, $n(B) = 6$, $B \subset A$: $n(A - B) =$ (a) 4 (b) 16 (c) 6 (d) 10
- If $(a + 3, b - 2) = (5, 1)$, then $(a, b) =$ (a) $(2, 3)$ (b) $(3, 2)$ (c) $(8, -1)$ (d) $(2, -1)$
- If $A = \{a, b, c\}$, $n(A \times A) =$ (a) 3 (b) 6 (c) 9 (d) 27
- Domain of $f(x) = \sqrt{x - 1}$ is (a) $\mathbb{R}$ (b) $[1, \infty)$ (c) $(1, \infty)$ (d) $(-\infty, 1]$
- Range of $f(x) = |x|$ is (a) $\mathbb{R}$ (b) $(0, \infty)$ (c) $[0, \infty)$ (d) $(-\infty, 0]$
- $[3.7] + [-3.7] =$ (a) 0 (b) -1 (c) 1 (d) 7
- If $f(x) = 2x + 3$, $f(f(1)) =$ (a) 5 (b) 13 (c) 10 (d) 8
- The graph of $f(x) = c$ (constant) is (a) a vertical line (b) a horizontal line (c) a parabola (d) a V-shape

18. Domain of $f(x) = (x^2 + 1)/(x^2 - 1)$ is (a) $\mathbb{R}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{-1, 1\}$ (d) $\mathbb{R} - \{0\}$
 19. The signum function at $x = -7$ equals (a) 7 (b) -7 (c) -1 (d) 0
 20. Range of $f(x) = 1/x^2$ is (a) $\mathbb{R}$ (b) $(0, \infty)$ (c) $[0, \infty)$ (d) $\mathbb{R} - \{0\}$

Answers: 1 c · 2 b · 3 c (2^5) · 4 b · 5 b · 6 b · 7 b (De Morgan) · 8 b · 9 c · 10 a · 11 a · 12 c · 13 b · 14 c · 15 b ($3 + (-4)$) · 16 b ($f(1) = 5, f(5) = 13$) · 17 b · 18 c · 19 c · 20 b.

Block 2 – Trigonometry (Q21–40)

1. $\pi/9$ radian in degrees: (a) 18° (b) 20° (c) 40° (d) 9°
 2. $\sin 300^\circ =$ (a) $1/2$ (b) $-1/2$ (c) $\sqrt{3}/2$ (d) $-\sqrt{3}/2$
 3. $\cos(-60^\circ) =$ (a) $-1/2$ (b) $1/2$ (c) $\sqrt{3}/2$ (d) $-\sqrt{3}/2$
 4. $\tan 135^\circ =$ (a) 1 (b) -1 (c) $\sqrt{3}$ (d) 0
 5. $\sec^2 x - \tan^2 x =$ (a) 0 (b) 1 (c) -1 (d) 2
 6. If $\sin x = 1/2$ and x is acute, $\cos x =$ (a) $1/2$ (b) $\sqrt{3}/2$ (c) $1/\sqrt{2}$ (d) 0
 7. $\sin(A + B) \sin(A - B) =$ (a) $\sin^2 A - \sin^2 B$ (b) $\cos^2 A - \cos^2 B$ (c) $\sin^2 A + \sin^2 B$ (d) 1
 8. $\cos 2x$ in terms of $\tan x$: (a) $2\tan x/(1 + \tan^2 x)$ (b) $(1 - \tan^2 x)/(1 + \tan^2 x)$ (c) $(1 + \tan^2 x)/(1 - \tan^2 x)$ (d) $2\tan x/(1 - \tan^2 x)$
 9. Minimum value of $\sin x \cos x$: (a) -1 (b) $-1/2$ (c) 0 (d) $1/2$
 10. $\sin 15^\circ =$ (a) $(\sqrt{3} - 1)/(2\sqrt{2})$ (b) $(\sqrt{3} + 1)/(2\sqrt{2})$ (c) $1/4$ (d) $\sqrt{3}/4$
 11. Period of $\tan 3x$: (a) π (b) $\pi/3$ (c) 3π (d) $2\pi/3$
 12. $\cos 3x =$ (a) $3\cos x - 4\cos^3 x$ (b) $4\cos^3 x - 3\cos x$ (c) $4\sin^3 x - 3\sin x$ (d) $\cos^3 x$
 13. $\cos 75^\circ - \cos 15^\circ =$ (a) $1/\sqrt{2}$ (b) $-1/\sqrt{2}$ (c) $\sqrt{3}/2$ (d) 0
 14. $\tan 75^\circ =$ (a) $2 + \sqrt{3}$ (b) $2 - \sqrt{3}$ (c) $\sqrt{3}$ (d) $1/\sqrt{3}$
 15. In quadrant IV, which is positive? (a) $\sin$ (b) $\tan$ (c) $\cos$ (d) $\cot$
 16. Radius of a circle in which a central angle of 60° intercepts an arc of 37.4 cm ($\pi = 22/7$): (a) 35.7 cm (b) 30 cm (c) 22 cm (d) 37.4 cm
 17. $1 - 2\sin^2(\pi/8) =$ (a) $1/\sqrt{2}$ (b) $1/2$ (c) $\sqrt{3}/2$ (d) 0
 18. Value of $\sin^2 60^\circ + \cos^2 60^\circ$ is (a) 1 (b) 0 (c) 2 (d) $1/2$
 19. If $\tan x = 1/\sqrt{3}$ and $x \in (\pi, 3\pi/2)$, $x =$ (a) $\pi/6$ (b) $7\pi/6$ (c) $5\pi/6$ (d) $4\pi/3$
 20. $2\cos^2 15^\circ - 1 =$ (a) $1/2$ (b) $\sqrt{3}/2$ (c) $1/\sqrt{2}$ (d) 0

Answers: 21 b · 22 d · 23 b · 24 b · 25 b · 26 b · 27 a · 28 b · 29 b ($\frac{1}{2} \sin 2x \geq -\frac{1}{2}$) · 30 a · 31 b · 32 b · 33 b ($-2 \sin 45^\circ \sin 30^\circ$) · 34 a · 35 c · 36 a · 37 a ($\cos \pi/4$) · 38 a · 39 b · 40 b ($\cos 30^\circ$).

Block 3 – Algebra (Q41–70)

1. $i^{2026} =$ (a) 1 (b) -1 (c) i (d) $-i$
 2. $(1 - i)^2 =$ (a) 2 (b) $-2i$ (c) $2i$ (d) 0
 3. $|3 - 4i| =$ (a) 7 (b) 1 (c) 5 (d) 25
 4. Conjugate of $i(1 + i)$ is (a) $-1 - i$ (b) $-1 + i$ (c) $1 - i$ (d) $1 + i$

5. If $z = 1 + i$, $z \bar{z} =$ (a) 2 (b) 0 (c) $2i$ (d) 1
6. Roots of $x^2 + 2 = 0$: (a) ± 2 (b) $\pm \sqrt{2} i$ (c) $\pm 2i$ (d) $\pm \sqrt{2}$
7. Solution of $2x - 3 > 7$: (a) $x > 5$ (b) $x < 5$ (c) $x > 2$ (d) $x \geq 5$
8. If $-x < -7$, then (a) $x < 7$ (b) $x > 7$ (c) $x = 7$ (d) $x < -7$
9. Solution of $|x - 1| < 3$: (a) $(-2, 4)$ (b) $(-4, 2)$ (c) $[-2, 4]$ (d) $(-\infty, 4)$
10. $10! / 8! =$ (a) 2 (b) 90 (c) 80 (d) 10
11. ${}^nC_r + {}^nC_{r-1} =$ (a) ${}^{n+1}C_r$ (b) ${}^nC_{r+1}$ (c) ${}^{n-1}C_r$ (d) ${}^{n+1}C_{r+1}$
12. Number of arrangements of "LEVEL": (a) 120 (b) 30 (c) 60 (d) 20
13. Ways to choose 2 from 6: (a) 30 (b) 12 (c) 15 (d) 36
14. Number of straight lines through 8 points, no 3 collinear: (a) 28 (b) 56 (c) 16 (d) 64
15. 4 boys and 3 girls in a row with girls together: (a) 5040 (b) 720 (c) 144 (d) 4320
16. If ${}^nC_3 = {}^nC_7$, then ${}^nC_2 =$ (a) 45 (b) 10 (c) 90 (d) 21
17. Number of terms in $(x + y)^{20}$: (a) 20 (b) 21 (c) 22 (d) 40
18. Coefficient of x^2 in $(1 + x)^5$: (a) 5 (b) 10 (c) 20 (d) 1
19. Sum of binomial coefficients of $(1 + x)^{10}$: (a) 100 (b) 1024 (c) 512 (d) 10
20. Last term of $(2x - 1)^7$: (a) 1 (b) -1 (c) $128x^7$ (d) -7
21. Middle term of $(1 + x)^4$: (a) $4x$ (b) $6x^2$ (c) $4x^3$ (d) x^2
22. Common ratio of $1/2, -1/4, 1/8, \dots$: (a) $1/2$ (b) $-1/2$ (c) 2 (d) -2
23. Sum to infinity of $1 + 1/3 + 1/9 + \dots$ (a) $3/2$ (b) 2 (c) 3 (d) $4/3$
24. If $1/4, x, 4$ are in G.P, $x =$ (a) ± 1 (b) 2 (c) 16 (d) $1/16$
25. Sum of first 10 natural numbers: (a) 45 (b) 55 (c) 100 (d) 50
26. $\sum k^2$ for $k = 1$ to 5: (a) 55 (b) 15 (c) 25 (d) 225
27. $\sum k^3$ for $k = 1$ to 4: (a) 64 (b) 100 (c) 30 (d) 36
28. A.M. of 7 and 13: (a) 10 (b) 20 (c) 6 (d) $\sqrt{91}$
29. If A.M. = G.M. for two positive numbers, then (a) $a > b$ (b) $a = b$ (c) $a < b$ (d) $ab = 1$
30. Sum of 5 terms of 1, 2, 4, ...: (a) 31 (b) 32 (c) 16 (d) 63

Answers: 41 b ($2026 = 4 \times 506 + 2$) · 42 b · 43 c · 44 a ($i + i^2 = -1 + i$, conjugate $-1 - i$) · 45 a · 46 b · 47 a · 48 b · 49 a · 50 b · 51 a · 52 b ($5!/(2!2!)$) · 53 c · 54 a · 55 b ($5! \times 3! = 720$) · 56 a ($n = 10$) · 57 b · 58 b · 59 b · 60 b · 61 b · 62 b · 63 a · 64 a · 65 b · 66 a · 67 b · 68 a · 69 b · 70 a.

Block 4 - Coordinate Geometry (Q71-90)

1. Distance between (0, 0) and (3, 4): (a) 7 (b) 5 (c) 25 (d) 1
2. Mid-point of (2, 4) and (6, 8): (a) (4, 6) (b) (8, 12) (c) (2, 2) (d) (3, 4)
3. Slope of a line making 45° with the x-axis: (a) 0 (b) 1 (c) $\sqrt{3}$ (d) -1
4. Line $y = 3$ is parallel to (a) y-axis (b) x-axis (c) $y = x$ (d) none
5. Slope of $4x - 2y + 7 = 0$: (a) 2 (b) -2 (c) $1/2$ (d) 4

6. Lines $y = 2x + 1$ and $y = 2x - 5$ are (a) perpendicular (b) parallel (c) intersecting (d) coincident
7. Equation of line with x-intercept 2 and y-intercept 3: (a) $3x + 2y = 6$ (b) $2x + 3y = 6$ (c) $x + y = 5$ (d) $3x - 2y = 6$
8. Perpendicular distance from (1, 1) to $x + y = 0$: (a) $\sqrt{2}$ (b) 2 (c) 1 (d) $1/\sqrt{2}$
9. Angle between $x = 2$ and $y = 3$: (a) 0° (b) 45° (c) 90° (d) 60°
10. Circle $x^2 + y^2 = 16$ has radius (a) 16 (b) 4 (c) 8 (d) 2
11. Centre of $(x - 2)^2 + (y + 3)^2 = 9$: (a) (2, -3) (b) (-2, 3) (c) (2, 3) (d) (3, 2)
12. Directrix of $y^2 = 8x$: (a) $x = 2$ (b) $x = -2$ (c) $y = 2$ (d) $y = -2$
13. Axis of $x^2 = 4y$: (a) x-axis (b) y-axis (c) $y = x$ (d) none
14. For $x^2/25 + y^2/16 = 1$, $c =$ (a) 3 (b) 9 (c) 41 (d) 4
15. Eccentricity of a circle: (a) 1 (b) 0 (c) > 1 (d) between 0 and 1
16. For the hyperbola $x^2/9 - y^2/16 = 1$, vertices are (a) $(\pm 3, 0)$ (b) $(\pm 4, 0)$ (c) $(\pm 5, 0)$ (d) $(0, \pm 3)$
17. Latus rectum of $x^2/9 + y^2/4 = 1$: (a) $8/3$ (b) $4/3$ (c) $9/2$ (d) 2
18. Distance of (1, -2, 3) from the origin: (a) $\sqrt{14}$ (b) 14 (c) 6 (d) $\sqrt{6}$
19. A point on the XZ-plane has (a) $x = 0$ (b) $y = 0$ (c) $z = 0$ (d) $x = z$
20. Number of octants: (a) 4 (b) 6 (c) 8 (d) 3

Answers: 71 b · 72 a · 73 b · 74 b · 75 a · 76 b · 77 a · 78 a ($|1 + 1|/\sqrt{2}$) · 79 c · 80 b · 81 a · 82 b · 83 b · 84 a · 85 b · 86 a · 87 a ($2 \times 4/3$) · 88 a · 89 b · 90 c.

Block 5 – Calculus, Statistics, Probability (Q91–120)

1. $\lim_{x \rightarrow 3} (2x + 1) =$ (a) 6 (b) 7 (c) 5 (d) 3
2. $\lim_{x \rightarrow 0} (1 - \cos x)/x =$ (a) 1 (b) 0 (c) $1/2$ (d) ∞
3. $\lim_{x \rightarrow 2} (x^3 - 8)/(x - 2) =$ (a) 8 (b) 12 (c) 4 (d) 6
4. $\lim_{x \rightarrow 0} \sin x/(2x) =$ (a) 2 (b) $1/2$ (c) 1 (d) 0
5. $\lim_{x \rightarrow 0} (a^x - 1)/x =$ (a) a (b) $\log a$ (c) 1 (d) 0
6. $d/dx(x^3) =$ (a) $3x$ (b) x^2 (c) $3x^2$ (d) 3
7. $d/dx(\tan x) =$ (a) $\sec x$ (b) $\sec^2 x$ (c) $-\operatorname{cosec}^2 x$ (d) $\cot x$
8. $d/dx(5) =$ (a) 5 (b) 0 (c) 1 (d) $5x$
9. $d/dx(\sqrt{x}) =$ (a) $2\sqrt{x}$ (b) $1/\sqrt{x}$ (c) $1/(2\sqrt{x})$ (d) $\sqrt{x}/2$
10. Derivative of $\sin x + \cos x$ at $x = \pi/4$: (a) $\sqrt{2}$ (b) 0 (c) 1 (d) -1
11. $d/dx(x^2 \sin x)$ at $x = 0$: (a) 0 (b) 1 (c) 2 (d) -1
12. If $y = 1/x$, dy/dx at $x = 2$: (a) $1/2$ (b) $-1/4$ (c) $1/4$ (d) $-1/2$
13. $d/dx(\sec x) =$ (a) $\sec x \tan x$ (b) $\sec^2 x$ (c) $-\sec x \tan x$ (d) $\tan^2 x$
14. Slope of tangent to $y = x^2$ at (1, 1): (a) 1 (b) 2 (c) 0 (d) 3
15. $d/dx[(x + 1)^2] =$ (a) $2(x + 1)$ (b) $x + 1$ (c) $2x$ (d) 2

16. Range of 5, 9, 2, 11, 7: (a) 9 (b) 11 (c) 6 (d) 2
17. Mean of first 5 natural numbers: (a) 2.5 (b) 3 (c) 15 (d) 5
18. Variance of 3, 3, 3, 3: (a) 3 (b) 9 (c) 0 (d) 1
19. If variance = 2.25, S.D. = (a) 1.5 (b) 2.25 (c) 5.06 (d) 0.5
20. S.D. of 2, 4, 6, 8, 10 is (a) 2 (b) $2\sqrt{2}$ (c) 8 (d) 4
21. Mean deviation about mean of 1, 2, 3: (a) 1 (b) $\frac{2}{3}$ (c) 0 (d) 2
22. Variance of first 9 natural numbers: (a) 7.5 (b) 9 (c) $\frac{80}{9}$ (d) $\frac{20}{3}$
23. Probability of a sure event: (a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2
24. A die is thrown: $P(\text{number} > 6) =$ (a) 1 (b) 0 (c) $\frac{1}{6}$ (d) $\frac{5}{6}$
25. $P(\text{an ace from a pack}) =$ (a) $\frac{1}{13}$ (b) $\frac{4}{13}$ (c) $\frac{1}{52}$ (d) $\frac{1}{4}$
26. Two coins: $P(\text{at least one tail}) =$ (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1
27. Two dice: $P(\text{sum} = 12) =$ (a) $\frac{1}{36}$ (b) $\frac{1}{12}$ (c) $\frac{1}{6}$ (d) $\frac{2}{36}$
28. If $P(A) = 0.6$, $P(B) = 0.5$, $P(A \cap B) = 0.2$, $P(A \cup B) =$ (a) 1.1 (b) 0.9 (c) 0.3 (d) 0.7
29. Number of outcomes when 3 coins are tossed: (a) 3 (b) 6 (c) 8 (d) 9
30. $P(\text{a red card from a pack}) =$ (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{13}$ (d) $\frac{3}{4}$

Answers: 91 b · 92 b · 93 b · 94 b · 95 b · 96 c · 97 b · 98 b · 99 c · 100 b (cos – sin = 0) · 101 a · 102 b · 103 a · 104 b · 105 a · 106 a · 107 b · 108 c · 109 a · 110 b (variance 8) · 111 b · 112 d $((81 - 1)/12)$ · 113 c · 114 b · 115 a · 116 c · 117 a · 118 b · 119 c · 120 b.

IMPORTANT PROOFS & QUICK REVISION

Mathematics · +2 1st Year

Proofs Frequently Asked in the Examination

1. Prove that $\sin^2 x + \cos^2 x = 1$ for all x .

Take a unit circle with centre O . Let the radius OP make angle x with the positive x -axis, and let $P = (a, b)$. By definition, $\cos x = a$ and $\sin x = b$. Drop a perpendicular PM to the x -axis; in right triangle OMP , $OM^2 + MP^2 = OP^2$, i.e. $a^2 + b^2 = 1$. Hence $\cos^2 x + \sin^2 x = 1$. This holds in all quadrants because squares remove the signs.

2. Prove $\cos(x + y) = \cos x \cos y - \sin x \sin y$.

On the unit circle take points $P_1(\cos x, \sin x)$, $P_2(\cos(x + y), \sin(x + y))$, $P_3(\cos(-y), \sin(-y))$ and $P_4(1, 0)$. Angle $P_1OP_3 = x + y = \text{angle } P_4OP_2$, so chords $P_1P_3 = P_2P_4$.

$$P_1P_3^2 = (\cos x - \cos y)^2 + (\sin x + \sin y)^2 = 2 - 2(\cos x \cos y - \sin x \sin y).$$

$$P_2P_4^2 = (1 - \cos(x + y))^2 + \sin^2(x + y) = 2 - 2\cos(x + y).$$

Equating: $\cos(x + y) = \cos x \cos y - \sin x \sin y$. Replacing y by $-y$ gives $\cos(x - y) = \cos x \cos y + \sin x \sin y$.

3. Deduce $\sin(x + y) = \sin x \cos y + \cos x \sin y$.

$$\begin{aligned} \sin(x + y) &= \cos(\pi/2 - (x + y)) = \cos((\pi/2 - x) - y) = \cos(\pi/2 - x) \cos y + \sin(\pi/2 - x) \sin y \\ \sin y &= \sin x \cos y + \cos x \sin y. \end{aligned}$$

4. Prove $\tan(x + y) = (\tan x + \tan y)/(1 - \tan x \tan y)$.

$$\tan(x + y) = \sin(x + y)/\cos(x + y) = (\sin x \cos y + \cos x \sin y)/(\cos x \cos y - \sin x \sin y).$$

Divide numerator and denominator by $\cos x \cos y$.

5. Prove the binomial theorem for positive integer n (by mathematical induction).

$$P(n): (a + b)^n = \sum_{r=0}^n {}^nC_r a^{n-r} b^r.$$

$$\text{For } n = 1: (a + b) = {}^1C_0 a + {}^1C_1 b \checkmark.$$

Assume $P(k)$ true. Then $(a + b)^{k+1} = (a + b)(a + b)^k = (a + b) \sum_{r=0}^k {}^kC_r a^{k-r} b^r$. Multiplying out and grouping terms with the same powers, the coefficient of $a^{k+1-r} b^r$ is ${}^kC_r + {}^kC_{r-1} = {}^{k+1}C_r$. Also first and last coefficients are $1 = {}^{k+1}C_0 = {}^{k+1}C_{k+1}$. Hence $P(k + 1)$ is true, and by induction $P(n)$ holds for all $n \in \mathbb{N}$.

6. Derive ${}^nP_r = n!/(n - r)!$.

Filling r places with n different things: the first place can be filled in n ways, the second in $(n - 1)$, ..., the r -th in $(n - r + 1)$. By the fundamental principle, ${}^nP_r = n(n - 1) \dots (n - r + 1) = n!/(n - r)!$.

7. Prove ${}^nC_r = n!/[r!(n - r)!]$.

Each selection of r things can be arranged in $r!$ ways, giving permutations. So ${}^nC_r \times r! = {}^nP_r \Rightarrow {}^nC_r = n!/[r!(n - r)!]$.

8. Prove ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

$$\text{LHS} = n!/[r!(n - r)!] + n!/[r!(n - r)!] = n!/[r!(n - r)!] \times [1/r + 1/(n - r + 1)] = n!/[r!(n - r)!] \times (n + 1)/[r(n - r + 1)] = (n + 1)!/[r!(n + 1 - r)!] = {}^{n+1}C_r.$$

9. Derive the sum of n terms of a G.P.

$S = a + ar + \dots + ar^{n-1}$. $rS = ar + ar^2 + \dots + ar^n$. Subtract: $S - rS = a - ar^n \Rightarrow S = a(1 - r^n)/(1 - r)$, $r \neq 1$. If $r = 1$, $S = na$.

10. Derive the distance of a point (x_1, y_1) from the line $Ax + By + C = 0$.

The line meets the axes at $Q(-C/A, 0)$ and $R(0, -C/B)$. Area of triangle $PQR = \frac{1}{2}|x_1(0 + C/B) - (C/A)(-C/B - y_1) + 0| = |C(Ax_1 + By_1 + C)|/(2|AB|)$. $QR = |C|\sqrt{(A^2 + B^2)}/|AB|$. Area = $\frac{1}{2} \times QR \times d \Rightarrow d = |Ax_1 + By_1 + C|/\sqrt{(A^2 + B^2)}$.

11. Derive the standard equation of the parabola $y^2 = 4ax$.

Let focus $S(a, 0)$ and directrix $x = -a$. For any point $P(x, y)$ on the parabola, $PS =$ distance from directrix: $\sqrt{(x - a)^2 + y^2} = |x + a|$. Squaring: $x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2 \Rightarrow y^2 = 4ax$.

12. Derive the standard equation of the ellipse.

Foci $F_1(-c, 0)$, $F_2(c, 0)$. $PF_1 + PF_2 = 2a$. $\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$.

Squaring and simplifying: $a\sqrt{(x-c)^2 + y^2} = a^2 - cx$. Squaring again: $(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$. Put $b^2 = a^2 - c^2$: $x^2/a^2 + y^2/b^2 = 1$.

13. Prove $\lim_{x \rightarrow 0} \sin x/x = 1$ (geometric idea).

In a unit circle with $0 < x < \pi/2$, compare areas: triangle OAP < sector OAP < triangle OAT, i.e. $\frac{1}{2} \sin x < \frac{1}{2} x < \frac{1}{2} \tan x$. Divide by $\frac{1}{2} \sin x$: $1 < x/\sin x < 1/\cos x$, i.e. $\cos x < \sin x/x < 1$. As $x \rightarrow 0$, $\cos x \rightarrow 1$, so $\sin x/x \rightarrow 1$ (sandwich theorem).

14. Prove the product rule $(uv)' = u'v + uv'$.

$[u(x+h)v(x+h) - u(x)v(x)]/h = [u(x+h)v(x+h) - u(x)v(x+h) + u(x)v(x+h) - u(x)v(x)]/h = v(x+h) \cdot [u(x+h) - u(x)]/h + u(x) \cdot [v(x+h) - v(x)]/h \rightarrow v u' + u v'$.

15. Show that variance $\sigma^2 = \Sigma x^2/n - \bar{x}^2$.

$\sigma^2 = \Sigma(x - \bar{x})^2/n = [\Sigma x^2 - 2\bar{x}\Sigma x + n\bar{x}^2]/n = \Sigma x^2/n - 2\bar{x}^2 + \bar{x}^2 = \Sigma x^2/n - \bar{x}^2$.

Chapter-wise Quick Revision Notes

Chapter	Must-know points (read the night before)
Sets	Roster/set-builder; $\varnothing \subset$ every set; 2^n subsets; De Morgan's laws; $n(A \cup B)$ formula; Venn diagrams for 2 and 3 sets.
Relations & Functions	$n(A \times B) = pq$; 2^{pq} relations; function = unique image; domain conditions (denominator $\neq 0$, $\sqrt{} \geq 0$, $\log > 0$); graphs of $ x $, $[x]$, signum, x^2 , $1/x$.
Trigonometry	$\pi = 180^\circ$; $l = r\theta$; ASTC signs; allied angles; compound-angle formulas; sum-to-product and product-to-sum; $2x$ and $3x$ formulas; max of $a \sin x + b \cos x = \sqrt{a^2 + b^2}$.
Complex numbers	$i^2 = -1$ and the 4-cycle; rationalise using conjugate; $ z $; Argand plane; quadratic formula with $D < 0$.
Linear inequalities	Reverse sign when multiplying/dividing by a negative; show answer on number line; word problems.
P & C	Counting principle; $n!$; nP_r , nC_r ; repeated letters; together/never together; committee problems.
Binomial	General term; middle term(s); term independent of x ; approximate values like $(99)^5$; divisibility proofs.
Sequences	A.P./G.P. n -th term and sum; infinite G.P. ($ r < 1$); A.M. $\geq$ G.M.; Σn , Σn^2 , Σn^3 ; $0.6 + 0.66 + \dots$ type series.
Straight lines	Slope; forms of line; angle between lines; parallel/perpendicular; distance of point from line; distance between parallel lines.
Conic sections	Circle centre/radius; parabola focus, directrix, LR; ellipse and hyperbola a , b , c , e , LR; identify orientation from which denominator is bigger / which term is positive.
3-D geometry	Octant signs; distance formula; collinearity and shapes by distances.
Limits & derivatives	Standard limits; factor/rationalise; first principle; sum, product, quotient rules; derivatives of trig functions.
Statistics	Range; M.D. about mean/median; variance and S.D. (direct and step-deviation); C.V. comparison.
Probability	Sample space; types of events; axioms; $P(\text{not } A)$, $P(A \text{ or } B)$; cards, dice, coins problems; combinations in probability.

Common Mistakes and How to Avoid Them

Mistake	Correct approach
Writing $\{1, 1, 2\}$ as having 3 elements	Repeated elements count once: $n = 2$.
Taking $\sqrt{(-4)} \cdot \sqrt{(-9)} = \sqrt{36} = 6$	$= 2i \cdot 3i = -6$.
Forgetting to reverse inequality sign	Multiply/divide by a negative $\Rightarrow$ reverse.
Using P where C is needed	Committees/selections $= C$; arrangements $= P$.
Middle term of $(a + b)^n$ as $n/2$ -th term	It is the $(n/2 + 1)$ -th term when n is even.
Applying S_∞ when $ r \geq 1$	Infinite sum exists only if $ r < 1$.
Writing slope of a vertical line as 0	Vertical line slope is undefined; horizontal line slope is 0.
Using $c^2 = a^2 + b^2$ for ellipse	Ellipse: $c^2 = a^2 - b^2$; hyperbola: $c^2 = a^2 + b^2$.
$\sin x/x \rightarrow 1$ when x is in degrees	The result needs x in radians.
Quotient rule order $(u'v - u'v')$	It is $(u'v - uv')/v^2$ — "derivative of top first".
Dividing by $n - 1$ in variance	CHSE formula divides by n (or N).
Adding probabilities of overlapping events	Subtract $P(A \cap B)$ unless they are mutually exclusive.

Chapter-wise Practice Exercises (Answers at the End)

Exercise A - Sets & Functions

- If $A = \{x : x \in \mathbb{N}, x \leq 6\}$ and $B = \{x : x \text{ is even}, x \leq 10\}$, find $A \cap B$ and $A - B$.
- Show that $A \cap (A \cup B) = A$.
- In a class of 35 students, 24 like cricket and 16 like football; each student likes at least one. How many like both?
- Find the domain of $f(x) = 1/(x^2 - 5x + 6)$.
- Find the range of $f(x) = 3 - x^2$.
- If $f(x) = x^3 - 1/x^3$, show that $f(x) + f(1/x) = 0$.

Exercise B - Trigonometry

- Find the radian measure of 520° .
- Find $\cos(-1710^\circ)$.
- Prove: $\sin(\pi + x)/\sin(-x) = 1$.
- Prove: $\sin 2x/(1 - \cos 2x) = \cot x$.
- If $\cos x = -1/2$ and x lies in quadrant III, find $\tan x$.
- Prove: $(\cos 4x + \cos 3x + \cos 2x)/(\sin 4x + \sin 3x + \sin 2x) = \cot 3x$.

Exercise C - Algebra

- Find the modulus of $(3 + 2i)(3 - 2i)$.
- Solve $x^2 - 2x + 3/2 = 0$.

3. Solve $3x + 8 > 2$ and show the solution set when $x \in \mathbb{R}$.
4. How many 4-digit numbers can be formed using 1-9 if repetition is not allowed?
5. In how many ways can 5 boys and 3 girls be seated in a row so that no two girls are together?
6. Expand $(1 - 2x)^5$.
7. Find the 4th term from the end of $(x - 1/x)^{12}$.
8. Find the sum to n terms of $0.3 + 0.33 + 0.333 + \dots$

Exercise D - Coordinate Geometry, Calculus, Statistics, Probability

1. Find the equation of the line through $(5, -2)$ parallel to $3x - 4y + 1 = 0$.
2. Find the circle with centre $(2, 3)$ touching the x -axis.
3. Find the focus and directrix of $y^2 = -8x$.
4. Evaluate $\lim_{x \rightarrow 0} \tan 2x / \sin 5x$.
5. Differentiate $x^2/(1 + x)$.
6. Find the S.D. of 1, 2, 3, 4, 5.
7. Find $P(\text{drawing a king or a queen})$ from a pack.
8. Find $P(\text{getting sum } 9)$ with two dice.

ANSWERS TO EXERCISES

A: 1. $A \cap B = \{2, 4, 6\}$, $A - B = \{1, 3, 5\}$ 2. Every element of A is in $A \cup B$ 3. 5 4. $\mathbb{R} - \{2, 3\}$ 5. $(-\infty, 3]$ 6. $f(1/x) = 1/x^3 - x^3$.

B: 1. $26\pi/9$ 2. 0 3. $-\sin x / (-\sin x) = 1$ 4. $2 \sin x \cos x / (2 \sin^2 x)$ 5. $\sqrt{3}$ 6. numerator = $\cos 3x(2\cos x + 1)$, denominator = $\sin 3x(2\cos x + 1)$.

C: 1. 13 2. $1 \pm i/\sqrt{2}$ 3. $x > -2$ 4. 3024 5. $5! \times {}^6P_3 = 14400$ 6. $1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$ 7. 4th from end = 10th from beginning = ${}^{12}C_9 x^3 (-1/x)^9 = -220/x^6$ 8. $(1/3)[n - (1 - 10^{-n})/9]$.

D: 1. $3x - 4y - 23 = 0$ 2. $(x - 2)^2 + (y - 3)^2 = 9$ 3. focus $(-2, 0)$, directrix $x = 2$ 4. $2/5$ 5. $(x^2 + 2x)/(1 + x)^2$ 6. $\sqrt{2}$ 7. $2/13$ 8. $4/36 = 1/9$.

HOTS & FINAL MOCK TEST

Mathematics · +2 1st Year · Higher-order thinking problems with solutions

HOTS Problems with Solutions

1. If A and B are two sets such that $n(A) = 2n(B)$ and $n(A \cup B) = 30$, $n(A \cap B) = 6$, find $n(A)$ and $n(B)$.

$$30 = 2n(B) + n(B) - 6 \Rightarrow 3n(B) = 36 \Rightarrow n(B) = 12, n(A) = 24.$$

2. Find the domain of $f(x) = \sqrt{x-1} + \sqrt{6-x}$.

$$x \geq 1 \text{ and } x \leq 6 \Rightarrow [1, 6].$$

3. Find the range of $f(x) = (x^2 - 9)/(x - 3)$, $x \neq 3$.

$$f(x) = x + 3 \text{ for } x \neq 3; \text{ it takes every real value except } 6. \text{ Range} = \mathbf{R - \{6\}}.$$

4. If $\tan x = n \tan y$, prove that $\tan^2(x - y) = (n - 1)^2 \tan^2 y / (1 + n \tan^2 y)^2$.

$$\tan(x - y) = (n \tan y - \tan y) / (1 + n \tan^2 y) = (n - 1) \tan y / (1 + n \tan^2 y). \text{ Square both sides.}$$

5. Prove: $\cos 20^\circ \cos 40^\circ \cos 80^\circ = 1/8$.

$$\text{Multiply and divide by } 2 \sin 20^\circ: 2 \sin 20^\circ \cos 20^\circ = \sin 40^\circ; \text{ then } 2 \sin 40^\circ \cos 40^\circ = \sin 80^\circ; \text{ then } 2 \sin 80^\circ \cos 80^\circ = \sin 160^\circ = \sin 20^\circ. \text{ So product} = \sin 20^\circ / (8 \sin 20^\circ) = 1/8.$$

6. If $\cos(x + y) = 4/5$, $\sin(x - y) = 5/13$ and x, y lie between 0 and $\pi/4$, find $\tan 2x$.

$$x + y \in (0, \pi/2): \sin(x + y) = 3/5, \tan(x + y) = 3/4. \quad x - y \in (-\pi/4, \pi/4) \text{ with positive sine: } \cos(x - y) = 12/13, \tan(x - y) = 5/12. \quad \tan 2x = \tan[(x + y) + (x - y)] = (3/4 + 5/12) / (1 - 15/48) = (14/12) / (33/48) = (7/6)(48/33) = \mathbf{56/33}.$$

7. Find the square root of $-8 - 6i$.

$(a + ib)^2 = -8 - 6i \Rightarrow a^2 - b^2 = -8, 2ab = -6$. Also $a^2 + b^2 = |-8 - 6i| = 10$. So $a^2 = 1, b^2 = 9; ab < 0 \Rightarrow (a, b) = (1, -3) \text{ or } (-1, 3)$. Square roots: $\pm(1 - 3i)$.

8. In how many ways can 6 persons be seated around a round table?

In circular arrangements one person's place is fixed: $(6 - 1)! = 120$.

9. How many triangles can be formed from 12 points of which 7 are collinear?

$${}^{12}C_3 - {}^7C_3 = 220 - 35 = \mathbf{185}.$$

10. Show that the coefficient of the middle term of $(1 + x)^{2n}$ equals the sum of the coefficients of the two middle terms of $(1 + x)^{2n-1}$.

Middle term of $(1 + x)^{2n}$: ${}^{2n}C_n$. Middle terms of $(1 + x)^{2n-1}$: ${}^{2n-1}C_{n-1}$ and ${}^{2n-1}C_n$. Their sum = ${}^{2n}C_n$ (by Pascal's rule).

11. If p, q, r are in G.P. and the equations $px^2 + 2qx + r = 0$ and $dx^2 + 2ex + f = 0$ have a common root, show that $d/p, e/q, f/r$ are in A.P.

$q^2 = pr$, so $px^2 + 2qx + r = 0$ has $D = 0$: equal roots $x = -q/p$. Put in the second: $d q^2/p^2 - 2eq/p + f = 0$. Divide by q^2/p (and use $q^2 = pr$): $d/p - 2e/q + f/r = 0 \Rightarrow 2(e/q) = d/p + f/r$. Hence A.P.

12. Find the equation of the line passing through (1, 2) such that the segment between the axes is bisected at this point.

If the intercepts are a and b , the mid-point $(a/2, b/2) = (1, 2) \Rightarrow a = 2, b = 4$. $x/2 + y/4 = 1 \Rightarrow \mathbf{2x + y = 4}$.

13. Find the equation of the circle which passes through the origin and cuts off intercepts 3 and 4 on the positive x - and y -axes.

Passes through $(0, 0), (3, 0), (0, 4)$: $x^2 + y^2 - 3x - 4y = 0$.

14. Evaluate $\lim_{x \rightarrow 0} (\sqrt{1 + x^2} - \sqrt{1 - x^2})/x^2$.

Rationalise: $2x^2/[x^2(\sqrt{1 + x^2} + \sqrt{1 - x^2})] \rightarrow 2/2 = 1$.

15. If $y = (1 - \tan x)/(1 + \tan x)$, find dy/dx .

$y = \tan(\pi/4 - x) \Rightarrow dy/dx = -\sec^2(\pi/4 - x)$.

16. Three dice are thrown. Find the probability of getting the same number on all three.

6 favourable out of 216: $1/36$.

Final Mock Test (Full Syllabus) with Answers

Time: 3 hours · Full marks: 80. Attempt all questions; internal choices are shown with OR.

Group A - Objective (1 × 12)

1. Number of proper subsets of $\{1, 2, 3, 4\}$.
2. Domain of $f(x) = \log(x + 2)$.
3. Value of $\cos 105^\circ$.
4. Value of $(1 + i)^8$.
5. Number of ways to arrange letters of "BOOK".
6. Coefficient of x^3 in $(1 + x)^7$.
7. Sum to infinity of $4 + 2 + 1 + \dots$
8. Slope of the line perpendicular to $y = 2x + 7$.
9. Radius of $x^2 + y^2 + 2x - 4y - 4 = 0$.
10. $\lim_{x \rightarrow 0} \sin 4x / \tan 2x$.
11. Variance of 5, 5, 5, 5, 5.
12. P(at least one six) when a die is thrown twice.

Answers: 1. 15 2. $(-2, \infty)$ 3. $(1 - \sqrt{3})/(2\sqrt{2})$ 4. 16 5. $4!/2! = 12$ 6. 35 7. 8 8. $-1/2$ 9. $\sqrt{(1 + 4 + 4)} = 3$ 10. 2 11. 0 12. $1 - 25/36 = 11/36$.

Group B - Short (2 × 8)

1. Draw a Venn diagram for $(A - B) \cup (B - A)$.
2. If $f(x) = x^2 - 2x$, find $f(x + 1) - f(x - 1)$.
3. Prove $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$.
4. Solve $-3 \leq 4 - 7x/2 \leq 18$.
5. Find n if ${}^nP_5 = 42 \cdot {}^nP_3$ ($n > 4$).
6. Find the distance between parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$.
7. Find the derivative of $1/(ax^2 + bx + c)$.
8. Find the mean deviation about the median of 3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21.

Answers: 13. Shade both circles except the common part. 14. $(x^2 - 1) - (x^2 - 4x + 3) = 4x - 4$. 15. See QB-3 Q8 in Exam Q&A. 16. $[-4, 2]$. 17. $(n - 3)(n - 4) = 42 \Rightarrow n = 10$. 18. $2/5$. 19. $-(2ax + b)/(ax^2 + bx + c)^2$. 20. $58/11 \approx 5.27$.

Group C - Short (4 × 6)

1. In a survey of 600 students, 150 drink tea, 225 coffee, 100 both. How many drink neither?
2. Prove: $(\cos 9x - \cos 5x)/(\sin 17x - \sin 3x) = -\sin 2x/\cos 10x$.
3. Find the real numbers x and y if $(x - iy)(3 + 5i) = -6 + 24i$.
4. Find the sum of the series $1 + 3 + 3^2 + \dots + 3^{n-1}$ and the least n for which it exceeds 1000.

5. Find the equation of the ellipse with $e = 3/4$ and foci on the y-axis, centre at origin, passing through (6, 4).
6. Find the derivative of $(x^5 - \cos x)/\sin x$.

Answers: 21. Tea or coffee = 275 $\Rightarrow$ neither = 325. 22. Numerator $-2 \sin 7x \sin 2x$; denominator $2 \cos 10x \sin 7x$. 23. $x = 3$, $y = -3$. 24. $(3^n - 1)/2 > 1000 \Rightarrow 3^n > 2001 \Rightarrow n = 7$. 25. $x^2/b^2 + y^2/a^2 = 1$, $b^2 = a^2(1 - 9/16) = 7a^2/16$. Then $36 \cdot 16/(7a^2) + 16/a^2 = 1 \Rightarrow (576/7 + 16) = a^2 \Rightarrow a^2 = 688/7$, $b^2 = 43$. Ellipse: $x^2/43 + 7y^2/688 = 1$. 26. $[(5x^4 + \sin x) \sin x - (x^5 - \cos x) \cos x]/\sin^2 x$.

Group D - Long (6 $\times$ 3)

1. Prove: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4 \cos x \cos 2x \sin 4x$. OR Find $\sin(x/2)$, $\cos(x/2)$, $\tan(x/2)$ if $\tan x = -4/3$, x in quadrant II.
2. A committee of 5 is to be chosen from 6 men and 4 women. In how many ways if (i) it has at least 2 women (ii) at most 2 women? OR Find the coefficient of x^5 in $(x + 3)^8$ and the term independent of x in $(2x - 1/x)^{10}$.
3. Calculate the mean and S.D. of: x : 60, 61, 62, 63, 64, 65, 66, 67, 68 with f : 2, 1, 12, 29, 25, 12, 10, 4, 5.

Answers: 27. See More Solved Examples (trig 5). OR: x in quadrant II $\Rightarrow x/2$ in quadrant I; $\cos x = -3/5$; $\sin(x/2) = \sqrt{(1 + 3/5)/2} = 2/\sqrt{5}$, $\cos(x/2) = 1/\sqrt{5}$, $\tan(x/2) = 2$.

28. (i) 2W: ${}^4C_2 {}^6C_3 = 120$; 3W: ${}^4C_3 {}^6C_2 = 60$; 4W: ${}^6C_1 = 6$. Total 186. (ii) 0W: 6; 1W: $4 \times 15 = 60$; 2W: 120. Total 186. OR: 1512 ; $T_{r+1} = {}^{10}C_r 2^{10-r} (-1)^r x^{10-2r}$, $r = 5$: $-252 \times 32 = -8064$.

29. Take $A = 64$: $y = x - 64 = -4, \dots, 4$. $N = 100$. $\Sigma fy = -8 - 3 - 24 - 29 + 0 + 12 + 20 + 12 + 20 = 0 \Rightarrow \text{mean} = 64$. $\Sigma fy^2 = 32 + 9 + 48 + 29 + 0 + 12 + 40 + 36 + 80 = 286 \Rightarrow \text{variance} = 2.86$, S.D. ≈ 1.69 .

PRACTICE WORKSHEETS

Mathematics · +2 1st Year · 14 chapter worksheets with answers

Each worksheet takes about 30 minutes. Write full working in your notebook and check with the answer key below each worksheet.

Worksheet 1 - Sets

- Write in roster form: $\{x : x \text{ is a two-digit natural number whose digits add to } 8\}$.
- Write in set-builder form: $\{2, 4, 8, 16, 32\}$.
- Which of these are empty: (a) $\{x : x \text{ is a natural number, } x < 5 \text{ and } x > 7\}$ (b) $\{x : x \text{ is a point common to two parallel lines}\}$ (c) $\{0\}$?
- Write all subsets of $\{a, b, c\}$.
- If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, find $A \cup B$, $A \cap B$, $A - B$, $B - A$.
- $U = \{1, \dots, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$. Find $(A \cup B)'$, $A' \cap B'$, $(A \cap B)'$ and $A' \cup B'$.
- In a town of 10,000 families, 40% buy newspaper A, 20% buy B, 10% buy C, 5% A and B, 3% B and C, 4% A and C, and 2% all three. Find the number of families buying (i) only A (ii) none.

Answers: 1. $\{17, 26, 35, 44, 53, 62, 71, 80\}$ 2. $\{x : x = 2^n, n \in \mathbb{N}, 1 \leq n \leq 5\}$ 3. (a) and (b) 4. $\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}$ 5. $\{1, \dots, 6\}, \{3,4\}, \{1,2\}, \{5,6\}$ 6. $\{5,7,9\}, \{5,7,9\}, \{1,3,5,6,7,8,9\}$, same 7. (i) $40 - 5 - 4 + 2 = 33\% = 3300$ (ii) At least one $= 40 + 20 + 10 - 5 - 3 - 4 + 2 = 60\% \Rightarrow$ none 4000.

Worksheet 2 - Relations and Functions

- If $A = \{-1, 1\}$, find $A \times A \times A$.
- If $P = \{a, b, c\}$ and $Q = \{r\}$, form $P \times Q$ and $Q \times P$. Are they equal?
- The Cartesian product $A \times A$ has 9 elements, two of which are $(-1, 0)$ and $(0, 1)$. Find A and the remaining elements.

4. Let $R = \{(x, y) : x + 2y = 8\}$ on N . Write R , its domain and range.
5. Which of these relations are functions? (i) $\{(2,1), (5,1), (8,1), (11,1)\}$ (ii) $\{(1,3), (1,5), (2,5)\}$ (iii) $\{(2,1), (4,2), (6,3), (8,4)\}$
6. Find the domain and range of $f(x) = -|x|$.
7. Find the domain of $f(x) = (x^2 + 3x + 5)/(x^2 - 5x + 4)$.
8. If $f(x) = x^2$, find $[f(1.1) - f(1)]/(1.1 - 1)$.

Answers: 1. $\{(-1,-1,-1), (-1,-1,1), (-1,1,-1), (-1,1,1), (1,-1,-1), (1,-1,1), (1,1,-1), (1,1,1)\}$ 2. $\{(a,r), (b,r), (c,r)\}$ and $\{(r,a), (r,b), (r,c)\}$; not equal 3. $A = \{-1, 0, 1\}$; others $(-1,-1), (-1,1), (0,-1), (0,0), (1,-1), (1,0), (1,1)$ 4. $\{(2,3), (4,2), (6,1)\}$; domain $\{2,4,6\}$; range $\{1,2,3\}$ 5. (i) and (iii) 6. $R; (-\infty, 0]$ 7. $R - \{1, 4\}$ 8. $0.21/0.1 = 2.1$.

Worksheet 3 - Trigonometric Functions

1. Find in radians the angle subtended at the centre of a circle of radius 15 cm by an arc of 10 cm.
2. Find the values of the other five ratios if $\cot x = 3/4$, x in quadrant III.
3. Find $\sin(-11\pi/3)$.
4. Prove: $\cos(\pi + x) \cos(-x)/[\sin(\pi - x) \cos(\pi/2 + x)] = \cot^2 x$.
5. Prove: $\sin^2(\pi/8 + x/2) - \sin^2(\pi/8 - x/2) = (1/\sqrt{2}) \sin x$.
6. Prove: $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2((x + y)/2)$.
7. Prove: $\cos 4x = 1 - 8 \sin^2 x \cos^2 x$.
8. Find the value of $\sin 22^\circ 30'$.

Answers: 1. $2/3$ rad 2. $\tan 4/3, \sin -4/5, \cos -3/5, \operatorname{cosec} -5/4, \sec -5/3$ 3. $\sqrt{3}/2$ 4. $\text{LHS} = (-\cos x)(\cos x)/[\sin x(-\sin x)] = \cot^2 x$ 5. Use $\sin^2 A - \sin^2 B = \sin(A + B) \sin(A - B) = \sin(\pi/4) \sin x$ 6. Expand: $2 + 2\cos(x + y) = 4\cos^2((x + y)/2)$ 7. $\cos 4x = 1 - 2\sin^2 2x$ 8. $\sin 22.5^\circ = \sqrt{(1 - \cos 45^\circ)/2} = \sqrt{(2 - \sqrt{2})/2}$.

Worksheet 4 - Complex Numbers & Quadratic Equations

1. Evaluate $[i^{18} + (1/i)^{25}]^3$.
2. Express $(1 - i)^4$ in $a + ib$ form.
3. Find the multiplicative inverse of $2 - 3i$.
4. Express $(5 + \sqrt{2}i)/(1 - \sqrt{2}i)$ in $a + ib$ form.
5. Solve $x^2 - x + 2 = 0$.
6. Solve $27x^2 - 10x + 1 = 0$.
7. If $|z| = 1$, show that $(z - 1)/(z + 1)$ is purely imaginary ($z \neq -1$). (Hint: put $z = \cos \theta + i \sin \theta$.)

Answers: 1. $(-1 - i)^3 = 2 - 2i$ 2. -4 3. $(2 + 3i)/13$ 4. $1 + 2\sqrt{2}i$ 5. $(1 \pm i\sqrt{7})/2$ 6. $(5 \pm i\sqrt{2})/27$ 7. $(z - 1)/(z + 1) = i \tan(\theta/2)$.

Worksheet 5 - Linear Inequalities

1. Solve $4x + 3 < 5x + 7$.
2. Solve $(x - 3)/(x - 5) > 0$.
3. Solve $37 - (3x + 5) \geq 9x - 8(x - 3)$.
4. Solve $7 \leq (3x + 11)/2 \leq 11$.
5. Solve the system $2x + 1 > x - 4$ and $3 - 4x > 2x - 9$.
6. A manufacturer has 600 litres of 12% acid solution. How many litres of 30% acid solution must be added so that the mixture has more than 15% but less than 18% acid?

Answers: 1. $x > -4$ 2. $x < 3$ or $x > 5$ 3. $x \leq 2$ 4. $[1, 11/3]$ 5. $(-5, 2)$ 6. more than 120 L and less than 300 L.

Worksheet 6 - Permutations and Combinations

1. Compute $8!/(6! 2!)$.
2. How many 4-digit numbers can be formed from 1, 2, 3, 4, 5 if (i) repetition is allowed (ii) not allowed?
3. Find the number of different signals that can be made by arranging at least 2 flags in order (one below the other) on a vertical staff, if 5 different flags are available.
4. How many words can be formed from the letters of "ALLAHABAD"?
5. Find the number of ways of selecting 9 balls from 6 red, 5 white and 5 blue balls if each selection has 3 balls of each colour.
6. How many 5-card combinations from a pack of 52 contain exactly one ace?
7. In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if discs of the same colour are identical?

Answers: 1. 28 2. 625; 120 3. ${}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5 = 20 + 60 + 120 + 120 = 320$ 4. $9!/(4!2!) = 7560$ 5. $20 \times 10 \times 10 = 2000$ 6. $4 \times {}^{48}C_4 = 4 \times 194580 = 778320$ 7. $9!/(4!3!2!) = 1260$.

Worksheet 7 - Binomial Theorem

1. Expand $(2x - 3)^6$ (first four terms).
2. Find $(a + b)^4 - (a - b)^4$ and hence evaluate $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$.
3. Find the 4th term of $(x - 2y)^{12}$.
4. Find the middle terms of $(x/3 + 9y)^{10}$.
5. Find the coefficient of x^4 in $(1 - x)^8$.
6. Evaluate $(0.99)^5$ correct to four decimal places using the first three terms.

Answers: 1. $64x^6 - 576x^5 + 2160x^4 - 4320x^3 + \dots$ 2. $8ab(a^2 + b^2)$; $8\sqrt{6} \times 5 = 40\sqrt{6}$ 3. $-1760x^9y^3$ 4. $T_6 = {}^{10}C_5(x/3)^5(9y)^5 = 252 \times 243 x^5y^5 = 61236x^5y^5$ 5. 70 6. $(1 - 0.01)^5 \approx 1 - 0.05 + 0.001 = 0.951$.

Worksheet 8 - Sequences and Series

- Find the 12th term of a G.P. whose 8th term is 192 and common ratio is 2.
- The 5th, 8th and 11th terms of a G.P. are p , q , s . Show that $q^2 = ps$.
- Find the sum of the G.P. 0.15, 0.015, 0.0015, ... to 20 terms.
- Find the sum of the first n terms of $1 + 4 + 13 + 40 + \dots$ (hint: differences are 3, 9, 27, ...).
- Show that $(a^{n+1} + b^{n+1})/(a^n + b^n)$ is the A.M. of a and b when $n = 0$ and the G.M. when $n = -1/2$.
- Find two positive numbers whose difference is 12 and whose A.M. exceeds G.M. by 2.

Answers: 1. $a = 192/2^7 = 1.5$; $a_{12} = 1.5 \times 2^{11} = 3072$ 2. $q = pr^3$, $s = pr^6$ 3. $(1/6)(1 - 10^{-20})$ 4. $a_k = (3^k - 1)/2$; $S_n = (3^{n+1} - 3)/4 - n/2$ 5. Direct substitution 6. $(\sqrt{a} - \sqrt{b})^2 = 4 \Rightarrow \sqrt{a} - \sqrt{b} = 2$ and $a - b = 12 \Rightarrow \sqrt{a} + \sqrt{b} = 6 \Rightarrow a = 16$, $b = 4$.

Worksheet 9 - Straight Lines

- Find the angle between the x-axis and the line joining $(3, -1)$ and $(4, -2)$.
- Find the equation of the line through $(-2, 3)$ with slope -4 .
- Find the equation of the line whose perpendicular distance from the origin is 4 and the angle between the perpendicular and the positive x-axis is 15° .
- Find the values of θ and p if $\sqrt{3}x + y + 2 = 0$ is written as $x \cos \theta + y \sin \theta = p$.
- Find the distance of $(-1, 1)$ from $12(x + 6) = 5(y - 2)$.
- Find the equation of the line through $(1, 2)$ and making angle 30° with the y-axis.

Answers: 1. 135° 2. $4x + y + 5 = 0$ 3. $x \cos 15^\circ + y \sin 15^\circ = 4$, i.e. $(\sqrt{6} + \sqrt{2})x + (\sqrt{6} - \sqrt{2})y = 16$ 4. $-\sqrt{3}x - y = 2 \Rightarrow$ divide by 2: $\cos \theta = -\sqrt{3}/2$, $\sin \theta = -1/2 \Rightarrow \theta = 210^\circ$, $p = 1$ 5. 5 6. Inclination 60° : $y - 2 = \sqrt{3}(x - 1)$.

Worksheet 10 - Conic Sections

- Find the circle with centre $(-a, -b)$ and radius $\sqrt{a^2 + b^2}$.
- Find the centre and radius of $2x^2 + 2y^2 - x = 0$.
- Find the parabola with vertex $(0, 0)$, focus $(0, 2)$.
- Find the parabola symmetric about the y-axis passing through $(2, -3)$.
- Find the ellipse with ends of major axis $(\pm 3, 0)$ and ends of minor axis $(0, \pm 2)$.
- Find the hyperbola with foci $(0, \pm 13)$ and conjugate axis of length 24.

Answers: 1. $x^2 + y^2 + 2ax + 2by + 2b^2 = 0$ 2. $(1/4, 0)$, $1/4$ 3. $x^2 = 8y$ 4. $x^2 = -4y/3$, i.e. $3x^2 = -4y$ 5. $x^2/9 + y^2/4 = 1$ 6. $b = 12$, $c = 13 \Rightarrow a = 5$: $y^2/25 - x^2/144 = 1$.

Worksheet 11 - 3-D Geometry

- Find the distance between $(-3, 7, 2)$ and $(2, 4, -1)$.
- Show that $(-2, 3, 5)$, $(1, 2, 3)$, $(7, 0, -1)$ are collinear.
- Find the equation of the set of points equidistant from $(1, 2, 3)$ and $(3, 2, -1)$.
- Name the octants of $(1, -2, 3)$ and $(-4, -2, -5)$.

Answers: 1. $\sqrt{43}$ 2. $AB + BC = AC$ ($\sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$) 3. $x - 2z = 0$ 4. IV and VII.

Worksheet 12 - Limits and Derivatives

- Evaluate $\lim_{x \rightarrow 1} (ax^2 + bx + c)/(cx^2 + bx + a)$, $a + b + c \neq 0$.
- Evaluate $\lim_{x \rightarrow 0} (\cos x)/(\pi - x)$.
- Evaluate $\lim_{x \rightarrow \pi/2} \tan 2x/(x - \pi/2)$.
- Find the derivative of $x^2 - 2$ at $x = 10$.
- Differentiate $(x - a)(x - b)$.
- Differentiate $(ax^2 + \sin x)(p + q \cos x)$.

Answers: 1. 1 2. $1/\pi$ 3. 2 4. 20 5. $2x - a - b$ 6. $(2ax + \cos x)(p + q \cos x) - q \sin x(ax^2 + \sin x)$.

Worksheet 13 - Statistics

- Find the mean deviation about the mean of 38, 70, 48, 40, 42, 55, 63, 46, 54, 44.
- Find the variance of 6, 7, 10, 12, 13, 4, 8, 12.
- The mean of 5 observations is 4.4 and variance 8.24. Three are 1, 2, 6. Find the others.
- The mean and variance of 7 observations are 8 and 16. If five are 2, 4, 10, 12, 14, find the remaining two.

Answers: 1. Mean 50; $\Sigma|x - 50| = 12 + 20 + 2 + 10 + 8 + 5 + 13 + 4 + 4 + 6 = 84$; M.D. = 8.4 2. Mean 9; $\Sigma(x - 9)^2 = 9 + 4 + 1 + 9 + 16 + 25 + 1 + 9 = 74$; variance 9.25 3. 4 and 9 4. $\text{Sum} = 56 \Rightarrow x + y = 14$; $\Sigma x^2 = 7(16 + 64) = 560 \Rightarrow x^2 + y^2 = 560 - 460 = 100 \Rightarrow xy = 48 \Rightarrow 6 \text{ and } 8$.

Worksheet 14 - Probability

- A coin is tossed three times. Write the sample space and $P(\text{at least two heads})$.
- One card is drawn from a pack. Find $P(\text{a black king})$, $P(\text{a spade or an ace})$.
- A box has 10 red, 20 blue, 30 green marbles. 5 are drawn. Find $P(\text{all blue})$.
- If $P(A) = 0.54$, $P(B) = 0.69$, $P(A \cap B) = 0.35$, find $P(A \cup B)$, $P(A' \cap B')$, $P(A \cap B')$.
- In an exam, 80% pass in English, 85% in Mathematics and 75% in both. What is the probability that a student passes in at least one subject?

Answers: 1. {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}; $4/8 = 1/2$ 2. $2/52 = 1/26$; $16/52 = 4/13$ 3. ${}^{20}C_5 / {}^{60}C_5 = 15504/5461512 = 34/11977$ 4. 0.88; 0.12; 0.19 5. $0.80 + 0.85 - 0.75 = 0.90$.

FORMULA BOOKLET

Mathematics · +2 1st Year · Everything on a few pages

Algebra Formulas

Item	Formula
$(a + b)^2, (a - b)^2$	$a^2 \pm 2ab + b^2$
$a^2 - b^2$	$(a + b)(a - b)$
$(a \pm b)^3$	$a^3 \pm 3a^2b + 3ab^2 \pm b^3$
$a^3 \pm b^3$	$(a \pm b)(a^2 \mp ab + b^2)$
Quadratic roots	$x = [-b \pm \sqrt{(b^2 - 4ac)}]/2a$; sum = $-b/a$; product = c/a
Nature of roots	$D > 0$ real distinct; $D = 0$ real equal; $D < 0$ complex conjugate
Laws of indices	$a^m a^n = a^{m+n}$; $a^m/a^n = a^{m-n}$; $(a^m)^n = a^{mn}$; $a^0 = 1$
Logarithms	$\log(mn) = \log m + \log n$; $\log(m/n) = \log m - \log n$; $\log m^n = n \log m$; $\log_a a = 1$

Sets, Functions, Complex Numbers

Item	Formula
Union (2 sets)	$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
Union (3 sets)	$n(A \cup B \cup C) = \Sigma n(A) - \Sigma n(A \cap B) + n(A \cap B \cap C)$
Subsets	2^n ; proper subsets $2^n - 1$
De Morgan	$(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$
Cartesian product	$n(A \times B) = n(A)n(B)$; relations $2^{n(A)n(B)}$
Powers of i	i, -1, -i, 1 (repeat every 4)
Modulus, conjugate	$ a + ib = \sqrt{a^2 + b^2}$; conj = $a - ib$; $z \bar{z} = z ^2$
Reciprocal	$1/z = \bar{z}/ z ^2$

Trigonometry

Item	Formula
Conversion	radian = degree $\times \pi/180$; $l = r\theta$
Pythagorean	$\sin^2 + \cos^2 = 1$; $1 + \tan^2 = \sec^2$; $1 + \cot^2 = \operatorname{cosec}^2$
Compound angles	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$; $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$; $\tan(A \pm B) = (\tan A \pm \tan B)/(1 \mp \tan A \tan B)$
Double angle	$\sin 2A = 2 \sin A \cos A$; $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$; $\tan 2A = 2\tan A/(1 - \tan^2 A)$
Triple angle	$\sin 3A = 3\sin A - 4\sin^3 A$; $\cos 3A = 4\cos^3 A - 3\cos A$; $\tan 3A = (3\tan A - \tan^3 A)/(1 - 3\tan^2 A)$
Half angle	$\sin(A/2) = \pm\sqrt{(1 - \cos A)/2}$; $\cos(A/2) = \pm\sqrt{(1 + \cos A)/2}$
Sum to product	$\sin C + \sin D = 2\sin((C+D)/2)\cos((C-D)/2)$; $\sin C - \sin D = 2\cos((C+D)/2)\sin((C-D)/2)$; $\cos C + \cos D = 2\cos((C+D)/2)\cos((C-D)/2)$; $\cos C - \cos D = -2\sin((C+D)/2)\sin((C-D)/2)$
Product to sum	$2\sin A \cos B = \sin(A+B) + \sin(A-B)$; $2\cos A \cos B = \cos(A+B) + \cos(A-B)$; $2\sin A \sin B = \cos(A-B) - \cos(A+B)$
Special values	$\sin 15^\circ = (\sqrt{3} - 1)/(2\sqrt{2})$; $\cos 15^\circ = (\sqrt{3} + 1)/(2\sqrt{2})$; $\tan 15^\circ = 2 - \sqrt{3}$; $\tan 75^\circ = 2 + \sqrt{3}$; $\tan 22.5^\circ = \sqrt{2} - 1$

Counting, Binomial, Sequences

Item	Formula
Permutations	${}^n P_r = n!/(n - r)!$; with repetitions $n!/(p! q! r!)$; circular $(n - 1)!$
Combinations	${}^n C_r = n!/r!(n - r)!$; ${}^n C_r = {}^n C_{n-r}$; ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$
Geometry counts	lines from n points ${}^n C_2$; triangles ${}^n C_3$; diagonals ${}^n C_2 - n$
Binomial	$T_{r+1} = {}^n C_r a^{n-r} b^r$; terms $n + 1$; $\Sigma {}^n C_r = 2^n$
A.P.	$a_n = a + (n - 1)d$; $S_n = (n/2)[2a + (n - 1)d]$
G.P.	$a_n = ar^{n-1}$; $S_n = a(r^n - 1)/(r - 1)$; $S_\infty = a/(1 - r)$, $ r < 1$
Means	$A = (a + b)/2$; $G = \sqrt{ab}$; $A \geq G$
Special sums	$\Sigma n = n(n+1)/2$; $\Sigma n^2 = n(n+1)(2n+1)/6$; $\Sigma n^3 = [n(n+1)/2]^2$

Coordinate Geometry

Item	Formula
Distance (2-D / 3-D)	$\sqrt{(\Delta x)^2 + (\Delta y)^2} / \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$
Section formula	$((mx_2 + nx_1)/(m + n), (my_2 + ny_1)/(m + n))$
Centroid	$((x_1 + x_2 + x_3)/3, (y_1 + y_2 + y_3)/3)$
Slope	$m = \tan\theta = (y_2 - y_1)/(x_2 - x_1) = -A/B$
Angle between lines	$\tan\theta = (m_1 - m_2)/(1 + m_1m_2) $
Line forms	$y - y_1 = m(x - x_1); y = mx + c; x/a + y/b = 1; x \cos\omega + y \sin\omega = p$
Distances	point-line $ Ax_1 + By_1 + C /\sqrt{A^2 + B^2}$; parallel lines $ C_1 - C_2 /\sqrt{A^2 + B^2}$
Circle	$(x - h)^2 + (y - k)^2 = r^2$; centre $(-g, -f)$, $r = \sqrt{g^2 + f^2 - c}$
Parabola $y^2 = 4ax$	focus $(a, 0)$; directrix $x = -a$; LR $4a$
Ellipse	$c^2 = a^2 - b^2$; $e = c/a$; LR $= 2b^2/a$; foci $(\pm c, 0)$
Hyperbola	$c^2 = a^2 + b^2$; $e = c/a$; LR $= 2b^2/a$; foci $(\pm c, 0)$

Calculus, Statistics, Probability

Item	Formula
Standard limits	$(x^n - a^n)/(x - a) \rightarrow na^{n-1}$; $\sin x/x \rightarrow 1$; $\tan x/x \rightarrow 1$; $(e^x - 1)/x \rightarrow 1$; $(a^x - 1)/x \rightarrow \log a$; $\log(1 + x)/x \rightarrow 1$
First principle	$f'(x) = \lim_{h \rightarrow 0} [f(x + h) - f(x)]/h$
Rules	$(u \pm v)' = u' \pm v'$; $(uv)' = u'v + uv'$; $(u/v)' = (u'v - uv')/v^2$
Derivatives	$x^n \rightarrow nx^{n-1}$; $\sin \rightarrow \cos$; $\cos \rightarrow -\sin$; $\tan \rightarrow \sec^2$; $\cot \rightarrow -\operatorname{cosec}^2$; $\sec \rightarrow \sec \tan$; $\operatorname{cosec} \rightarrow -\operatorname{cosec} \cot$
Mean deviation	$\Sigma x - \bar{x} /n$; $\Sigma f x - \bar{x} /N$
Variance	$\Sigma(x - \bar{x})^2/n = \Sigma x^2/n - \bar{x}^2$; step deviation $h^2[\Sigma fy^2/N - (\Sigma fy/N)^2]$
C.V.	$(\sigma/\bar{x}) \times 100$
Probability	$P(E) = n(E)/n(S)$; $P(E') = 1 - P(E)$; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

PRACTICE & REVISION

Mathematics · +2 1st Year (Class XI)

Mathematics – Extra Solved Problems

1. If $A = \{x : x \text{ is a letter of the word "ASSASSINATION"}\}$, find $n(A)$.

Distinct letters: A, S, I, N, T, O $\rightarrow n(A) = 6$.

2. Show that $A \cup B = A \cap B$ implies $A = B$.

Let $x \in A \Rightarrow x \in A \cup B = A \cap B \Rightarrow x \in B$. So $A \subset B$. Similarly $B \subset A$. Hence $A = B$.

3. Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ be a linear function $f(x) = ax + b$. Find a and b .

$f(0) = b = -1$. $f(1) = a + b = 1 \Rightarrow a = 2$. So $f(x) = 2x - 1$.

4. Find the range of $f(x) = 1/(2 - \cos 3x)$.

$\cos 3x \in [-1, 1] \Rightarrow 2 - \cos 3x \in [1, 3] \Rightarrow f(x) \in [1/3, 1]$.

5. Prove: $\cos(\pi/4 + x) + \cos(\pi/4 - x) = \sqrt{2} \cos x$.

Using $\cos A + \cos B = 2\cos^{1/2}(A+B)\cos^{1/2}(A-B)$: $= 2\cos(\pi/4)\cos x = 2(1/\sqrt{2})\cos x = \sqrt{2} \cos x$.

6. If $\tan x = 3/4$ and x in Q III, find $\sin(x/2)$, $\cos(x/2)$, $\tan(x/2)$.

In Q III, $\cos x = -4/5$. $x/2$ lies in Q II ($\pi/2 < x/2 < 3\pi/4$). $\sin^2(x/2) = (1 - \cos x)/2 = 9/10 \Rightarrow \sin(x/2) = 3/\sqrt{10}$. $\cos^2(x/2) = (1 + \cos x)/2 = 1/10 \Rightarrow \cos(x/2) = -1/\sqrt{10}$. $\tan(x/2) = -3$.

7. Find the modulus of $(1 + i)/(1 - i) - (1 - i)/(1 + i)$.

$(1 + i)/(1 - i) = (1 + i)^2/2 = 2i/2 = i$; the second $= -i$. Difference $= 2i$; modulus $= 2$.

8. If $(x + iy)^3 = u + iv$, show that $u/x + v/y = 4(x^2 - y^2)$.

$(x + iy)^3 = x^3 - 3xy^2 + i(3x^2y - y^3)$. $u/x = x^2 - 3y^2$; $v/y = 3x^2 - y^2$. Sum = $4x^2 - 4y^2$. Proved.

9. Solve the system $2x - 1 > x + 3$ and $3x + 2 < 2x + 10$.

First: $x > 4$. Second: $x < 8$. Solution **(4, 8)**.

10. How many 4-digit numbers divisible by 5 can be formed from 0, 1, 3, 5, 7, 9 without repetition?

Last digit 0: remaining 3 places from 5 digits: $5 \times 4 \times 3 = 60$. Last digit 5: first digit cannot be 0 $\rightarrow$ 4 choices, then $4 \times 3 \times 4 \times 3 = 48$. Total = **108**.

11. In how many ways can 5 girls and 3 boys be seated in a row so that no two boys sit together?

Arrange girls: $5! = 120$. Gaps between/around girls = 6; choose and arrange 3 boys: ${}^6P_3 = 120$. Total = **14400**.

12. Find the coefficient of a^5b^7 in $(a - 2b)^{12}$.

$T_{r+1} = {}^{12}C_r a^{12-r} (-2b)^r$; $r = 7$. Coefficient = ${}^{12}C_7 (-2)^7 = 792 \times (-128) =$
-101376.

13. Find the sum to n terms of $0.6 + 0.66 + 0.666 + \dots$

$= (6/9)[0.9 + 0.99 + \dots] = (2/3)[(1 - 0.1) + (1 - 0.01) + \dots] = (2/3)[n - (1/10)(1 - 10^{-n})/(9/10)] =$
 $(2/3)[n - (1 - 10^{-n})/9]$.

14. Find the equation of the line through (3, 4) whose intercepts are equal.

$x/a + y/a = 1 \Rightarrow x + y = a$. Passing through (3, 4): $a = 7$. **$x + y = 7$** .

15. Find the equation of the ellipse with major axis on the x-axis, passing through (4, 3) and (-1, 4).

$x^2/a^2 + y^2/b^2 = 1$. Put $p = 1/a^2$, $q = 1/b^2$: $16p + 9q = 1$, $p + 16q = 1$. Solving: $p = 7/247$, $q = 15/247$. Ellipse: **$7x^2 + 15y^2 = 247$** .

16. Evaluate $\lim_{x \rightarrow 0} (\cos 2x - 1)/(\cos x - 1)$.

$= (-2\sin^2 x)/(-2\sin^2(x/2)) = [\sin x/\sin(x/2)]^2 \rightarrow (x/(x/2))^2 =$ **4**.

17. Differentiate $y = \sin x/(1 + \cos x)$ and simplify.

$y' = [\cos x(1 + \cos x) + \sin^2 x]/(1 + \cos x)^2 = (1 + \cos x)/(1 + \cos x)^2 =$ **$1/(1 + \cos x)$** .

18. The mean and variance of 8 observations are 9 and 9.25. If six of them are 6, 7, 10, 12, 12, 13, find the other two.

Sum = 72 $\Rightarrow x + y = 72 - 60 = 12$. $\Sigma x^2 = 8(9.25 + 81) = 722$. Known squares: $36 + 49 + 100 + 144 + 144 + 169 = 642 \Rightarrow x^2 + y^2 = 80$. With $x + y = 12$: $xy = (144 - 80)/2 = 32 \Rightarrow x, y = 4 \text{ and } 8$.

19. Three letters are dictated to three persons and envelopes are addressed. Letters are put in envelopes at random. Find the probability that all letters are in the correct envelopes.

Total arrangements $3! = 6$; only 1 correct. $P = 1/6$.

20. From a group of 8 boys and 5 girls a committee of 5 is formed. Find the probability that it has exactly 3 girls.

${}^5C_3 \times {}^8C_2 / {}^{13}C_5 = 10 \times 28/1287 = 280/1287$.

Mathematics - One-Page Formula Sheet (Class XI)

Topic	Key formulas
Sets	$n(A \cup B) = n(A) + n(B) - n(A \cap B)$; subsets = 2^n ; $(A \cup B)' = A' \cap B'$
Relations	$n(A \times B) = pq$; relations = 2^{pq}
Trig	$\pi \text{ rad} = 180^\circ$; $l = r\theta$; $\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$; $\sin 2x$, $\cos 2x$, $\tan 2x$; $\sin 3x = 3\sin x - 4\sin^3 x$; $\cos 3x = 4\cos^3 x - 3\cos x$
Complex	$i^2 = -1$; $ z = \sqrt{a^2 + b^2}$; $1/z = \bar{z}/ z ^2$
P & C	${}^nP_r = n!/(n-r)!$; ${}^nC_r = n!/[r!(n-r)!]$; $n!/(p!q!)$
Binomial	$T_{r+1} = {}^nC_r a^{n-r} b^r$; terms $n+1$
A.P./G.P.	$a + (n-1)d$; $(n/2)[2a + (n-1)d]$; ar^{n-1} ; $a(r^n - 1)/(r - 1)$; $a/(1-r)$; $A \geq G$
Lines	$m = (y_2 - y_1)/(x_2 - x_1)$; $\tan \theta = (m_2 - m_1)/(1 + m_1 m_2) $; $d = ax_1 + by_1 + c /\sqrt{a^2 + b^2}$
Conics	Circle centre $(-g, -f)$, $r = \sqrt{g^2 + f^2 - c}$; parabola $y^2 = 4ax$, LR $4a$; ellipse $c^2 = a^2 - b^2$; hyperbola $c^2 = a^2 + b^2$; $e = c/a$; LR $= 2b^2/a$
3-D	$d = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$
Limits	$(x^n - a^n)/(x - a) \rightarrow na^{n-1}$; $\sin x/x \rightarrow 1$; $(e^x - 1)/x \rightarrow 1$
Derivatives	$(uv)' = u'v + uv'$; $(u/v)' = (u'v - uv')/v^2$; $(\sin x)' = \cos x$; $(\cos x)' = -\sin x$; $(\tan x)' = \sec^2 x$
Statistics	M.D. = $\Sigma x - \bar{x} /n$; $\sigma^2 = \Sigma x^2/n - \bar{x}^2$; C.V. = $\sigma/\bar{x} \times 100$
Probability	$P(A') = 1 - P(A)$; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Model Question Paper - Mathematics Class XI (with hints)

Time: 3 hours · Full marks: 80

Section A – Very short (1 mark each)

1. Write the set $\{x : x \in \mathbb{N}, x^2 < 20\}$ in roster form. Ans: $\{1, 2, 3, 4\}$
2. Find the value of $\sin 765^\circ$. Ans: $\sin 45^\circ = 1/\sqrt{2}$
3. If ${}^nC_9 = {}^nC_8$, find ${}^nC_{17}$. Ans: $n = 17$, so 1
4. Find the slope of a line perpendicular to $2x + 3y = 5$. Ans: $3/2$
5. Find the eccentricity of $x^2/25 + y^2/9 = 1$. Ans: $4/5$
6. Evaluate $\lim_{x \rightarrow 0} \sin 5x/x$. Ans: 5
7. Find the number of terms in $(2x + 3y)^9$. Ans: 10
8. If $P(A) = 0.35$, find $P(\text{not } A)$. Ans: 0.65

Section B – Short (2-4 marks each)

1. Prove that $\cos 2x \cos(x/2) - \cos 3x \cos(9x/2) = \sin 5x \sin(5x/2)$. Hint: write each product as half the sum of two cosines ($2\cos A \cos B = \cos(A+B) + \cos(A-B)$), then use $\cos C - \cos D$.
2. Solve $x^2 - x + 2 = 0$. Ans: $(1 \pm i\sqrt{7})/2$
3. Find the 4th term in the expansion of $(x - 2y)^{12}$. Ans: $-1760x^9y^3$
4. Find the equation of the circle with centre (2, 2) passing through (4, 5). Ans: $(x-2)^2 + (y-2)^2 = 13$
5. Find the derivative of $(x^2 + 1) \cos x$. Ans: $2x \cos x - (x^2 + 1) \sin x$
6. Find the variance of the first 10 natural numbers. Ans: $(n^2 - 1)/12 = 99/12 = 8.25$

Section C – Long (6 marks each)

1. Find the mean deviation about the median for: 3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21.
Sorted: 3,3,4,5,7,9,10,12,18,19,21; median = 9; $\sum |x-9| = 6+6+5+4+2+0+1+3+9+10+12 = 58$; M.D. = $58/11 \approx 5.27$
2. Find the foci, vertices, eccentricity and latus rectum of $16x^2 - 9y^2 = 576$. $x^2/36 - y^2/64 = 1$; $a = 6$, $b = 8$, $c = 10$; foci $(\pm 10, 0)$; vertices $(\pm 6, 0)$; $e = 5/3$; LR = $64/3$
3. In how many ways can the letters of the word ALLAHABAD be arranged? In how many of them do the vowels occupy even positions? $9!/(4!2!) = 7560$. Vowels (4 A's) in 4 even places: 1 way; consonants L,L,H,B,D in 5 odd places: $5!/2! = 60$.

Mathematics – More Trigonometry & Limit Practice**1. Prove: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4 \cos x \cos 2x \sin 4x$.**

$$(\sin 7x + \sin x) + (\sin 5x + \sin 3x) = 2 \sin 4x \cos 3x + 2 \sin 4x \cos x = 2 \sin 4x (\cos 3x + \cos x) = 2 \sin 4x \cdot 2 \cos 2x \cos x = 4 \cos x \cos 2x \sin 4x.$$

2. Prove: $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4 \sin^2((x - y)/2)$.

$$\begin{aligned} \text{Expand: } \cos^2 x + \cos^2 y - 2 \cos x \cos y + \sin^2 x + \sin^2 y - 2 \sin x \sin y &= 2 - 2 \cos(x - y) \\ &= 2 \cdot 2 \sin^2((x - y)/2) = 4 \sin^2((x - y)/2). \end{aligned}$$

3. Find the value of $\cos(-1710^\circ)$.

$\cos$ is even: $\cos 1710^\circ = \cos(4 \times 360^\circ + 270^\circ) = \cos 270^\circ = \mathbf{0}$.

4. Evaluate $\lim_{x \rightarrow 0} (ax + x \cos x)/(b \sin x)$.

$= x(a + \cos x)/(b \sin x) \rightarrow (a + 1)/b$.

5. Evaluate $\lim_{x \rightarrow \pi/2} \tan 2x/(x - \pi/2)$.

Put $y = x - \pi/2 \rightarrow 0$: $\tan(2y + \pi) = \tan 2y$. So $\lim \tan 2y/y = \mathbf{2}$.

6. Find $\lim_{x \rightarrow 1} f(x)$, where $f(x) = x + 2$ ($x \leq 1$) and $f(x) = x - 2$ ($x > 1$).

LHL = $1 + 2 = 3$; RHL = $1 - 2 = -1$. LHL $\neq$ RHL, so **the limit does not exist**.

7. Differentiate from first principles $f(x) = 1/x$.

$[1/(x+h) - 1/x]/h = -h/[hx(x+h)] = -1/[x(x+h)] \rightarrow \mathbf{-1/x^2}$.

8. Differentiate $x^4(5 \sin x - 3 \cos x)$.

$4x^3(5 \sin x - 3 \cos x) + x^4(5 \cos x + 3 \sin x)$.

Last-Minute Exam Tips

Mathematics

- In trigonometry proofs, start from the more complicated side and use one identity per line.
- For degree-radian conversion, always multiply by $\pi/180$ (degree $\rightarrow$ radian) or $180/\pi$ (radian $\rightarrow$ degree).
- In P & C, first decide whether order matters (P) or not (C); handle repeated letters by dividing by their factorials.
- In binomial questions, write $T_{r+1} = {}^nC_r a^{n-r} b^r$ first, then find r .
- When dividing an inequality by a negative number, reverse the sign.
- For conics, find a , b , c first and then write foci, vertices, e and latus rectum in a neat list.
- Probability answers must lie between 0 and 1 — check your answer.

THE NIGHT BEFORE THE EXAM

Revise only formula boxes, name reactions, grammar rules and your own mistake list. Sleep well — a fresh mind solves problems faster than a tired one. Carry admit card, two pens, pencil, scale, eraser and a geometry box. Good luck!