



Physics Made Simple

Easy notes + solved problems for every chapter

CHSE Odisha · Class XI & XII · Syllabus 2023–24

28 chapters

230 solved problems

Detailed theory

56 long answers

30 diagrams

224 MCQs

1-mark Q&A

How to use these notes

💡 **Coloured box** – idea in simple words 📖 **More theory** – derivations, facts & applications

📌 **Yellow box** – formulas to remember

📚 **Theory in detail** – concepts in paragraphs 📐 **Diagrams** ⚡ **Quick 1-mark questions**

🎯 **Remember in one line** – quick revision ✅ **MCQs** with answer key

📝 **Long answer questions** – 5-mark derivations and explanations with full answers

✍️ **Solved problems** – read the steps, then cover them and try yourself ✓ green
– final answer

Contents & Marks Plan

Class XI

1	Units and Measurements
2	Motion in a Straight Line
3	Motion in a Plane
4	Laws of Motion
5	Work, Energy and Power
6	System of Particles and Rotational Motion
7	Gravitation
8	Mechanical Properties of Solids
9	Mechanical Properties of Fluids
10	Thermal Properties of Matter
11	Thermodynamics
12	Kinetic Theory of Gases
13	Oscillations
14	Waves

Units (Class XI theory, 70 marks)	Marks
I-IV: Measurement, Kinematics, Laws of Motion, Work-Energy	23
V-VI: Rotational Motion, Gravitation	17
VII-IX: Bulk Matter, Thermodynamics, Kinetic Theory	20
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Class XII

1	Electric Charges and Fields
2	Electrostatic Potential and Capacitance
3	Current Electricity
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5	Magnetism and Matter
6	Electromagnetic Induction
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8	Electromagnetic Waves
9	Ray Optics and Optical Instruments
10	Wave Optics
11	Dual Nature of Radiation and Matter
12	Atoms
13	Nuclei
14	Semiconductor Electronics

Units (Class XII theory, 70 marks)	Marks
I-II: Electrostatics, Current Electricity	16
III-IV: Magnetism, EMI & AC	17
V-VI: EM Waves, Optics	18
VII-VIII: Dual Nature, Atoms & Nuclei	12
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Practical exam: 30 marks in each class (experiment 15, record 4, conceptual question 5, viva 6).



Class XI

Mechanics · Properties of Matter · Heat · Oscillations & Waves

01 Units and Measurements

Class XI · Unit I

In simple words

- **Measurement** means comparing a quantity with a fixed standard called a **unit**.
- **SI system** has 7 base (fundamental) units: metre (m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol), candela (cd).
- **Derived units** are made from base units, e.g. newton = kg m s^{-2} .
- **Significant figures** = reliable digits + first doubtful digit. In multiplication/division, keep the answer to the *least* number of significant figures.
- **Dimensions** show how a quantity depends on M, L, T. Used to (i) check equations, (ii) convert units, (iii) find relations between quantities.

Remember in one line

- | | |
|---|---|
| 1 A unit is a fixed standard used to measure a quantity. | 2 SI has 7 base units; all others are derived from them. |
| 3 Dimensions tell how a quantity is built from M, L and T. | 4 Every term in a correct equation has the same dimensions. |
| 5 Least count is the smallest reading an instrument can give. | 6 Errors always add up when quantities are combined. |

Theory in detail

Why do we need standard units?

Imagine measuring a table with your hand span and your friend measuring it with theirs — the answers differ. A **standard unit** is agreed upon by everyone, never changes, can be copied easily and is available everywhere. That is why the world uses the SI system. Today the metre is defined using the speed of light, the second using caesium atom vibrations and the kilogram using Planck's constant, so they never change.

Least count and instruments

The **least count** is the smallest value an instrument can measure. A metre scale has least count 1 mm, a Vernier calliper 0.1 mm (0.01 cm) and a screw gauge 0.01 mm. Smaller least count means higher precision. Vernier calliper least count = $1 \text{ MSD} - 1 \text{ VSD}$; screw gauge least count = $\text{pitch} / \text{number of circular divisions}$. Always check and correct the **zero error** before measuring.

Types of errors

Systematic errors push results in one direction — caused by faulty instruments (zero error), wrong method, or personal habit (parallax). They can be reduced by better instruments and techniques. **Random errors** happen irregularly (small changes in temperature, voltage) and are reduced by taking many readings and averaging. **Gross errors** are simple mistakes like wrong recording.

Fundamental and derived quantities

Seven quantities are chosen as **fundamental** because they are independent of each other: length, mass, time, electric current, temperature, amount of substance and luminous intensity. All other quantities such as speed, force, pressure and energy are **derived** from them. Two supplementary units are the radian (plane angle) and steradian (solid angle).

How to write an answer with units

Always write the unit symbol after a number with a space (5 m, not 5m). Symbols are never pluralised (10 kg, not 10 kgs) and have no full stop. Unit names named after scientists are written in small letters (newton) but their symbols use capitals (N). Large and small values use prefixes: kilo (10^3), mega (10^6), milli (10^{-3}), micro (10^{-6}), nano (10^{-9}).

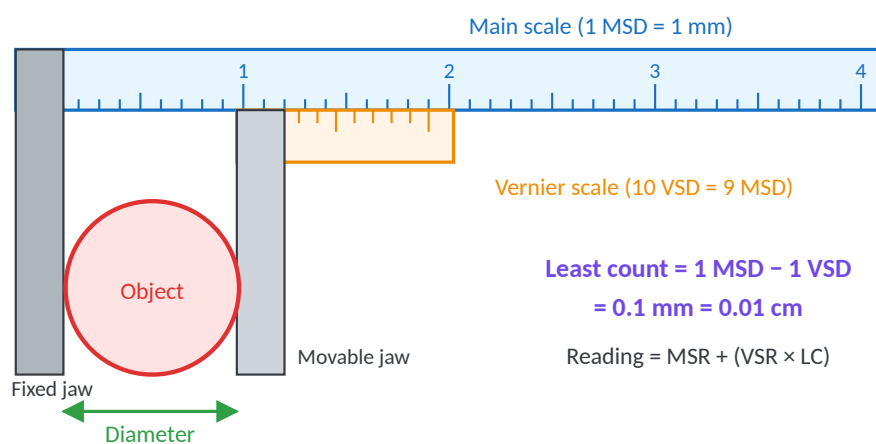


Fig. 1.1: Vernier caliper: measuring the diameter of a sphere

More theory - explained

- **Errors in measurement:** Absolute error $\Delta a = |a_{\text{true}} - a_i|$. Mean absolute error = average of all absolute errors. Relative error = $\Delta a/a$; Percentage error = $(\Delta a/a) \times 100$.
- **Combination of errors:** (i) $Z = A \pm B \Rightarrow \Delta Z = \Delta A + \Delta B$. (ii) $Z = AB$ or $A/B \Rightarrow \Delta Z/Z = \Delta A/A + \Delta B/B$. (iii) $Z = A^n \Rightarrow \Delta Z/Z = n(\Delta A/A)$. Errors always **add up**.
- **Accuracy vs Precision:** Accuracy = how close to the true value. Precision = how fine the measurement is (least count). A reading can be precise but not accurate.
- **Rules for significant figures:** (1) All non-zero digits count. (2) Zeros between non-zero digits count. (3) Leading zeros (0.00...) do not count. (4) Trailing zeros after a decimal point count (2.50 has 3 SF).
- **Rounding off:** if the dropped digit > 5 increase by 1; < 5 leave it; = 5, make the preceding digit even ($2.745 \rightarrow 2.74$, $2.735 \rightarrow 2.74$).
- **Limitations of dimensional analysis:** cannot find dimensionless constants (like 2π), cannot derive equations with sin, log or exp, and cannot handle equations with sums of terms.
- **Big and small units:** 1 light year = 9.46×10^{15} m; 1 AU = 1.496×10^{11} m; 1 parsec = 3.08×10^{16} m; 1 Å = 10^{-10} m; 1 fermi = 10^{-15} m.

Key formulas

$$[\text{Force}] = [\text{M L T}^{-2}]$$

$$[\text{Energy}] = [\text{M L}^2 \text{T}^{-2}]$$

$$[\text{Pressure}] = [\text{M L}^{-1} \text{T}^{-2}]$$

Principle of homogeneity: every term must have same dimensions

Quick 1-mark questions

SI unit of luminous intensity?

candela (cd)

Give one dimensionless quantity.

Strain / angle / refractive index

Which is bigger: 1 light year or 1 parsec?

1 parsec (≈ 3.26 ly)

Solved problems (6)

Q1 Check whether $v = u + at$ is dimensionally correct.

1. $[v] = [\text{L T}^{-1}]$, $[u] = [\text{L T}^{-1}]$

2. $[at] = [\text{L T}^{-2}][\text{T}] = [\text{L T}^{-1}]$

3. All terms have the same dimensions.

✓ **Answer: The equation is dimensionally correct.**

Q2 Using dimensions, find how the time period T of a simple pendulum depends on length l and g .

1. Let $T = k l^a g^b$

2. $[T] = [\text{L}]^a [\text{L T}^{-2}]^b = \text{L}^{a+b} \text{T}^{-2b}$

3. Compare: $-2b = 1 \Rightarrow b = -\frac{1}{2}$; $a + b = 0 \Rightarrow a = \frac{1}{2}$

✓ **Answer: $T = k\sqrt{l/g}$ (experiment gives $k = 2\pi$)**

Q3 How many significant figures are there in (a) 0.00230 (b) 2.0400 (c) 6.02×10^{23} ?

1. (a) leading zeros do not count; 2, 3 and trailing 0 count

2. (b) all digits count (zeros between and after decimal)

3. (c) powers of 10 do not count

✓ **Answer: (a) 3 (b) 5 (c) 3**

Q4 Convert 1 newton into dyne using dimensions.

1. $[F] = \text{M L T}^{-2}$; $n_2 = n_1(M_1/M_2)(L_1/L_2)(T_1/T_2)^{-2}$

2. $= 1 \times (1 \text{ kg}/1 \text{ g})(1 \text{ m}/1 \text{ cm})(1)^{-2} = 1000 \times 100$

✓ **Answer: $1 \text{ N} = 10^5 \text{ dyne}$**

Q5 $Z = A^2B/C$. Errors in A, B, C are 1%, 2% and 3%. Find the maximum % error in Z.

1. $\Delta Z/Z = 2(\Delta A/A) + \Delta B/B + \Delta C/C$

2. $= 2(1) + 2 + 3$

✓ Answer: 7%

Q6 Time periods measured: 2.63, 2.56, 2.42, 2.71, 2.80 s. Find the mean value, mean absolute error and % error.

1. Mean $= 13.12/5 = 2.624 \approx 2.62$ s

2. Absolute errors: 0.01, 0.06, 0.20, 0.09, 0.18 $\Rightarrow$ mean $= 0.54/5 \approx 0.11$ s

3. % error $= (0.11/2.62) \times 100$

✓ Answer: $T = (2.62 \pm 0.11)$ s, about 4%

Long answer questions (5 marks)

L1 What is dimensional analysis? State its uses and limitations with one example each.

Answer:

1. Dimensions of a quantity are the powers of M, L, T (and A, K, mol, cd) in its unit. Writing a quantity in this form is its dimensional formula, e.g. $[F] = [M L T^{-2}]$.
2. **Use 1 - Checking equations:** by the principle of homogeneity, every term must have the same dimensions. Example: $s = ut + \frac{1}{2}at^2 \Rightarrow [L] = [L] + [L] \checkmark$.
3. **Use 2 - Converting units:** $n_2 = n_1[M_1/M_2]^a[L_1/L_2]^b[T_1/T_2]^c$. Example: $1 \text{ J} = 10^7 \text{ erg}$.
4. **Use 3 - Deriving relations:** e.g. $T = k\sqrt{l/g}$ for a simple pendulum.
5. **Limitations:** cannot find numerical constants (2π); fails for trigonometric, log and exponential relations; cannot derive relations with more than 3 unknown quantities in mechanics; a dimensionally correct equation may still be wrong.

★ Dimensional analysis is a quick check, but not a full proof of an equation.

L2 Explain significant figures. Give the rules for arithmetic operations with examples.

Answer:

1. Significant figures are the reliably known digits plus the first uncertain digit. More SF = more precise measurement.
2. **Counting rules:** non-zero digits count; zeros between them count; leading zeros do not; trailing zeros after a decimal point count; changing units does not change SF ($2.30 \text{ m} = 230 \text{ cm} = 2.30 \times 10^2 \text{ cm}$, all 3 SF).
3. **Multiplication/division:** answer keeps as many SF as the least precise number. $4.237 \times 2.51 = 10.6$ (3 SF).
4. **Addition/subtraction:** answer keeps as many decimal places as the number with fewest decimal places. $436.32 + 227.2 + 0.301 = 663.8$.
5. **Rounding:** done only in the final answer to avoid adding errors at each step.

★ Significant figures tell the reader how precise a measurement is.

✓ Multiple choice questions (MCQs)

1. The SI unit of luminous intensity is

- (a) lumen (b) candela
(c) lux (d) watt

2. Dimensional formula of force is

- (a) $[M L T^{-1}]$ (b) $[M L^2 T^{-2}]$
(c) $[M L T^{-2}]$ (d) $[M L^{-1} T^{-2}]$

3. Which pair has the same dimensions?

- (a) Work and power (b) Work and torque
(c) Force and pressure (d) Momentum and energy

4. Number of significant figures in 0.00450 is

- (a) 2 (b) 3
(c) 5 (d) 6

5. 1 light year is a unit of

- (a) time (b) speed
(c) distance (d) intensity

6. Least count of a common Vernier calliper is

- (a) 1 mm (b) 0.1 mm
(c) 0.01 mm (d) 0.001 mm

7. If error in radius is 2%, error in volume of a sphere is

- (a) 2% (b) 4%
(c) 6% (d) 8%

8. Which is dimensionless?

- (a) Strain (b) Stress
(c) Velocity (d) Force

Answer key: 1-(b) Candela is one of the 7 base units. | 2-(c) $F = ma = \text{kg}\cdot\text{m}/\text{s}^2$. | 3-(b) Both are $[M L^2 T^{-2}]$. | 4-(b) Leading zeros don't count; 4, 5, 0 do. | 5-(c) Distance light travels in one year. | 6-(b) 1 MSD – 1 VSD = 0.1 mm. | 7-(c) $V \propto r^3 \Rightarrow 3 \times 2\%$. | 8-(a) Strain = length/length.

02 Motion in a Straight Line

Class XI • Unit II

In simple words

- **Distance** is total path (scalar); **displacement** is shortest straight gap from start to end (vector).
- **Velocity** = rate of change of displacement, $v = dx/dt$. **Acceleration** = rate of change of velocity, $a = dv/dt$.
- Slope of **x-t graph** = velocity. Slope of **v-t graph** = acceleration. Area under v-t graph = displacement.
- For uniform acceleration we use the three equations of motion.
- Free fall: $a = g = 9.8 \text{ m/s}^2$ (take 10 m/s^2 for easy sums) acting downward.

Remember in one line

- | | |
|--|---|
| 1 Distance is total path; displacement is shortest gap with direction. | 2 Velocity is how fast position changes; acceleration is how fast velocity changes. |
| 3 Slope of x-t graph = velocity; slope of v-t graph = acceleration. | 4 Area under v-t graph = displacement. |
| 5 Use $v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$ only for constant a. | 6 In free fall, all bodies have the same acceleration g. |

Theory in detail

Frame of reference

Motion is always **relative**. To describe motion we choose a reference point (origin), a set of axes and a clock — together called a **frame of reference**. A passenger sitting in a moving train is at rest with respect to the train but moving with respect to the platform. So we must always say 'with respect to what'.

Uniform and non-uniform motion

In **uniform motion** a body covers equal distances in equal intervals of time — its velocity is constant and acceleration is zero. In **non-uniform motion** the velocity changes, so there is acceleration. Acceleration can be positive (speeding up) or negative (retardation). If velocity and acceleration are in the same direction, speed increases; if opposite, speed decreases.

Using calculus for motion

When velocity keeps changing, average values are not enough. Differentiation gives the rate of change: $v = dx/dt$ and $a = dv/dt = d^2x/dt^2$. Integration does the reverse — it adds up small changes: $x = \int v \, dt$ and $v = \int a \, dt$. Geometrically, differentiation = slope of a graph and integration = area under a graph.

Position-time graph reading

On an x-t graph, a straight line means constant velocity; a steeper line means faster motion. A horizontal line means the body is at rest. A curve bending upward shows increasing speed

(positive acceleration); bending downward shows slowing down. A line going down means the body moves back towards the origin (negative velocity).

Motion under gravity

Near Earth's surface all bodies fall with the same acceleration $g \approx 9.8 \text{ m/s}^2$ if air resistance is ignored — a feather and a coin fall together in vacuum. For a body thrown up: time of ascent = time of descent, speed on return = speed of throw, and at the highest point velocity is zero but acceleration is still g downward.

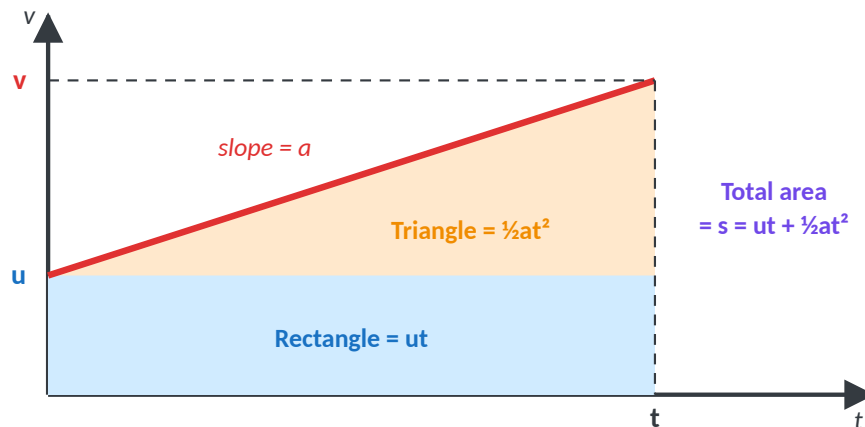


 Fig. 2.1: v-t graph for uniform acceleration: slope gives a , area gives s

More theory - explained

- **Average speed vs average velocity:** average speed = total distance / total time; average velocity = displacement / time. Average speed is always $\geq |\text{average velocity}|$.
- **Instantaneous velocity** $v = \lim_{\Delta t \rightarrow 0} \Delta x / \Delta t = dx/dt$. Useful calculus: $d(t^n)/dt = n t^{n-1}$, $\int t^n dt = t^{n+1}/(n+1)$.
- **Equations from v-t graph:** slope = $(v - u)/t = a$ gives $v = u + at$. Area of the trapezium under the graph = $\frac{1}{2}(u + v)t$ gives $s = ut + \frac{1}{2}at^2$.
- **Graph shapes:** uniform velocity $\rightarrow$ x-t straight line; uniform acceleration $\rightarrow$ x-t parabola and v-t straight inclined line; at rest $\rightarrow$ x-t horizontal line.
- **Relative velocity:** $v_{AB} = v_A - v_B$. Same direction $\rightarrow$ subtract speeds; opposite $\rightarrow$ add speeds.
- **Galileo's law of odd numbers:** in free fall from rest, distances covered in successive equal seconds are in ratio 1 : 3 : 5 : 7 ...
- **Stopping distance** = $u^2/2a$, so it is proportional to the square of speed (double speed $\rightarrow$ 4 times stopping distance).

Key formulas

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s_n \text{ (nth second)} = u + a(2n-1)/2$$

Quick 1-mark questions

Can velocity be zero while acceleration is not?	Yes, at the top of a vertical throw
Area under a-t graph gives?	Change in velocity
Slope of x-t graph gives?	Velocity

Solved problems (7)

Q1 A car starts from rest with acceleration 2 m/s^2 . Find its velocity and distance after 5 s.

1. $u = 0, a = 2 \text{ m/s}^2, t = 5 \text{ s}$
2. $v = u + at = 0 + 2 \times 5 = 10 \text{ m/s}$
3. $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2 \times 25 = 25 \text{ m}$

✓ Answer: $v = 10 \text{ m/s}, s = 25 \text{ m}$

Q2 A ball is thrown upward at 20 m/s . Find maximum height and total time in air. ($g = 10 \text{ m/s}^2$)

1. At top $v = 0$. Using $v^2 = u^2 - 2gH \Rightarrow H = u^2/2g = 400/20$
2. Time to rise $t = u/g = 20/10 = 2 \text{ s}$
3. Time up = time down, so total = 2×2

✓ Answer: $H = 20 \text{ m}, \text{ total time} = 4 \text{ s}$

Q3 $x = 3t^2 + 2t + 5$ (in m). Find velocity at $t = 2 \text{ s}$ and acceleration.

1. $v = dx/dt = 6t + 2 \Rightarrow$ at $t = 2 \text{ s}, v = 14 \text{ m/s}$
2. $a = dv/dt = 6 \text{ m/s}^2$ (constant)

✓ Answer: $v = 14 \text{ m/s}, a = 6 \text{ m/s}^2$

Q4 A body starts from rest with $a = 2 \text{ m/s}^2$. Find the distance covered in the 5th second.

1. $s_n = u + a(2n - 1)/2 = 0 + 2(9)/2$

✓ Answer: 9 m

Q5 A car at 20 m/s brakes with retardation 5 m/s^2 . Find the stopping distance. What if speed is doubled?

1. $s = u^2/2a = 400/10 = 40 \text{ m}$
2. At 40 m/s : $s = 1600/10 = 160 \text{ m}$ (4 times)

✓ Answer: $40 \text{ m}; 160 \text{ m}$

Q6 A stone is dropped from a 45 m tower. Find time to reach ground and final speed. ($g = 10$)

1. $t = \sqrt{2h/g} = \sqrt{(90/10)} = 3 \text{ s}$

2. $v = gt = 10 \times 3$

✓ Answer: $t = 3 \text{ s}$, $v = 30 \text{ m/s}$

Q7 Two trains move at 10 m/s and 15 m/s. Find their relative speed if moving (a) same direction (b) opposite.

1. (a) $15 - 10$

2. (b) $15 + 10$

✓ Answer: (a) 5 m/s (b) 25 m/s

Long answer questions (5 marks)

L1 Derive the three equations of uniformly accelerated motion using the velocity-time graph.

Answer:

- Take a body with initial velocity u at $t = 0$ and velocity v at time t , with constant acceleration a . Its v - t graph is a straight line from $A(0, u)$ to $B(t, v)$.
- First equation:** acceleration = slope = $(v - u)/t \Rightarrow v = u + at$.
- Second equation:** displacement = area under graph = area of rectangle ($u \times t$) + area of triangle $\frac{1}{2}(v - u)t = ut + \frac{1}{2}(at)t \Rightarrow s = ut + \frac{1}{2}at^2$.
- Third equation:** area = trapezium = $\frac{1}{2}(u + v)t$. Put $t = (v - u)/a$: $s = (v + u)(v - u)/2a \Rightarrow v^2 = u^2 + 2as$.

★ These three equations hold only when acceleration is constant.

L2 Derive the equations of motion using calculus.

Answer:

- $a = dv/dt \Rightarrow dv = a dt$. Integrate from u to v and 0 to t : $v - u = at \Rightarrow v = u + at$.
- $v = dx/dt \Rightarrow dx = (u + at)dt$. Integrate from 0 to s : $s = ut + \frac{1}{2}at^2$.
- $a = dv/dt = (dv/dx)(dx/dt) = v dv/dx \Rightarrow v dv = a dx$. Integrate: $(v^2 - u^2)/2 = as \Rightarrow v^2 = u^2 + 2as$.

★ Calculus gives the same equations and also works when a changes with time.

✓ Multiple choice questions (MCQs)

1. Slope of a velocity-time graph gives

- (a) distance (b) displacement
(c) acceleration (d) speed

2. Area under a v - t graph gives

- (a) acceleration (b) displacement
(c) velocity (d) force

3. A body moving in a circle returns to start. Its displacement is

- (a) $2\pi r$ (b) πr
(c) zero (d) r

4. A ball is thrown up. At the highest point

- (a) $v = 0, a = 0$ (b) $v = 0, a = g$
(c) $v = g, a = 0$ (d) $v \neq 0, a = 0$

5. A car starts from rest with $a = 4 \text{ m/s}^2$. Speed after 5 s is

- (a) 10 m/s (b) 20 m/s
(c) 25 m/s (d) 40 m/s

6. x - t graph of a body at rest is

- (a) parallel to time axis (b) parallel to x axis
(c) a parabola (d) inclined straight line

7. Distances fallen from rest in 1st, 2nd, 3rd seconds are in ratio

- (a) 1:2:3 (b) 1:4:9
(c) 1:3:5 (d) 1:1:1

8. If speed is doubled, stopping distance becomes

- (a) same (b) double
(c) four times (d) half

Answer key: 1-(c) $a = \Delta v / \Delta t$. | 2-(b) Area = $v \times t$. | 3-(c) Start and end points are same. | 4-(b) Gravity still acts. | 5-(b) $v = at = 20$. | 6-(a) x does not change. | 7-(c) Galileo's law of odd numbers. | 8-(c) $s = u^2 / 2a$.

03 Motion in a Plane

Class XI • Unit II

In simple words

- **Scalar** has only magnitude (mass, time). **Vector** has magnitude and direction (force, velocity).
- Vectors are added by **triangle / parallelogram law**. A vector can be split into components: $A_x = A \cos\theta$, $A_y = A \sin\theta$.
- **Dot product** gives a scalar ($A \cdot B = AB \cos\theta$); **cross product** gives a vector ($|A \times B| = AB \sin\theta$).
- **Projectile**: horizontal velocity stays constant, vertical motion has acceleration g . Path is a parabola.
- **Uniform circular motion**: speed constant, but direction changes, so there is a centripetal acceleration towards the centre.

Remember in one line

- | | |
|---|--|
| 1 Vectors have size and direction; scalars have size only. | 2 Split a vector into x and y parts to add easily. |
| 3 Dot product gives a scalar; cross product gives a vector. | 4 In a projectile, horizontal speed stays the same. |
| 5 Maximum range is at 45° . | 6 In circular motion, acceleration points to the centre. |

Theory in detail

Why vectors?

Some quantities need a direction to make full sense. 'Walk 5 km' is incomplete; 'walk 5 km towards north' tells you exactly where you will reach. Such quantities are **vectors**. Two vectors are **equal** only if both magnitude and direction are the same. Multiplying a vector by a positive number changes its length; by a negative number also reverses its direction.

Resolution of a vector

Any vector can be broken into two perpendicular parts called **rectangular components**. For a vector A at angle θ with the x-axis: $A_x = A \cos\theta$ and $A_y = A \sin\theta$, so $A = A_x \hat{i} + A_y \hat{j}$, $|A| = \sqrt{A_x^2 + A_y^2}$ and $\tan\theta = A_y/A_x$. Adding vectors becomes easy: add the x-parts and y-parts separately.

Motion in a plane with constant acceleration

With constant acceleration a , the vector forms of the equations hold: $v = u + at$ and $r = ut + \frac{1}{2}at^2$. They can be split into two independent one-dimensional motions along x and y. This idea — **independence of perpendicular motions** — is the key to solving projectile problems.

Addition and subtraction of vectors

Triangle law: place the tail of B at the head of A; the arrow from the tail of A to the head of B is $A + B$. **Parallelogram law**: draw A and B from the same point; the diagonal is their sum. Vector

addition is commutative ($A + B = B + A$) and associative. **Subtraction:** $A - B = A + (-B)$, where $-B$ has the same size but opposite direction.

Relative velocity in two dimensions

The velocity of A as seen by B is $v_{AB} = v_A - v_B$. Example: rain falling vertically appears to fall slanting to a person walking; he must tilt his umbrella forward. A swimmer crossing a river must head slightly upstream to reach the point exactly opposite; to cross in the shortest time he should swim straight across and accept drifting downstream.

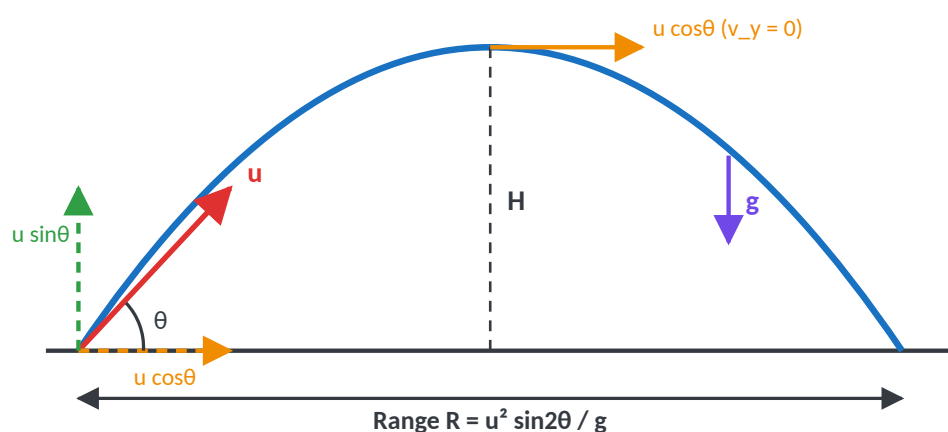


Fig. 3.1: Projectile motion: horizontal velocity stays constant, vertical velocity changes

More theory - explained

- **Resultant of two vectors** at angle θ : $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$, $\tan \beta = \frac{B \sin \theta}{A + B \cos \theta}$.
Maximum = $A + B$ ($\theta = 0^\circ$), minimum = $|A - B|$ ($\theta = 180^\circ$).
- **Unit vector** $\hat{A} = A/|A|$ has magnitude 1 and shows direction. $\hat{i}, \hat{j}, \hat{k}$ are along x, y, z.
- **Dot product** is commutative: $\hat{i} \cdot \hat{i} = 1, \hat{i} \cdot \hat{j} = 0$. **Cross product** is anti-commutative: $A \times B = -B \times A$; $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}; \hat{i} \times \hat{i} = 0$.
- **Projectile path:** $x = (u \cos \theta)t, y = (u \sin \theta)t - \frac{1}{2}gt^2 \Rightarrow y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$, which is a parabola.
- **Range facts:** maximum range at 45° , $R_{\max} = u^2/g$. Angles θ and $(90^\circ - \theta)$ give the same range.
At the highest point velocity = $u \cos \theta$ (horizontal) and acceleration = g (down).
- **Horizontal projection** from height h : time $t = \sqrt{2h/g}$, horizontal distance = $u\sqrt{2h/g}$.
- **Circular motion terms:** angular velocity $\omega = \theta/t, v = r\omega, T = 2\pi/\omega, f = 1/T$. Centripetal acceleration $a = v^2/r = \omega^2 r$ is always towards the centre.

Key formulas

$$\text{Range } R = \frac{u^2 \sin 2\theta}{g}$$

$$\text{Max height } H = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{Time of flight } T = \frac{2u \sin \theta}{g}$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

Quick 1-mark questions

Can two vectors add to zero?	Yes, if equal and opposite
Acceleration at the highest point of a projectile?	g , vertically down
Angle between $\mathbf{A} \times \mathbf{B}$ and $\mathbf{A}$?	90°

Solved problems (7)

Q1 A ball is projected at 20 m/s at 30° to the horizontal. Find T , H and R . ($g = 10 \text{ m/s}^2$)

1. $T = 2u \sin\theta / g = 2 \times 20 \times 0.5 / 10 = 2 \text{ s}$
2. $H = u^2 \sin^2\theta / 2g = 400 \times 0.25 / 20 = 5 \text{ m}$
3. $R = u^2 \sin 2\theta / g = 400 \times \sin 60^\circ / 10 = 40 \times 0.866$

✓ Answer: $T = 2 \text{ s}$, $H = 5 \text{ m}$, $R \approx 34.6 \text{ m}$

Q2 If $\mathbf{A} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{B} = \mathbf{i} + 2\mathbf{j}$, find $|\mathbf{A}|$, $\mathbf{A} \cdot \mathbf{B}$ and $\mathbf{A} \times \mathbf{B}$.

1. $|\mathbf{A}| = \sqrt{3^2 + 4^2} = 5$
2. $\mathbf{A} \cdot \mathbf{B} = (3)(1) + (4)(2) = 11$
3. $\mathbf{A} \times \mathbf{B} = (3 \times 2 - 4 \times 1) \mathbf{k} = 2\mathbf{k}$

✓ Answer: $|\mathbf{A}| = 5$, $\mathbf{A} \cdot \mathbf{B} = 11$, $\mathbf{A} \times \mathbf{B} = 2\mathbf{k}$

Q3 Forces of 3 N and 4 N act at 90° . Find the resultant and its direction.

1. $R = \sqrt{9 + 16} = 5 \text{ N}$
2. $\tan\beta = 4/3 \Rightarrow \beta \approx 53^\circ$ with the 3 N force

✓ Answer: 5 N at 53°

Q4 Find the angle between $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$.

1. $\mathbf{A} \cdot \mathbf{B} = (1)(1) + (1)(-1) = 0$
2. $\cos\theta = 0$

✓ Answer: $\theta = 90^\circ$

Q5 A ball is thrown at 30 m/s. Find the maximum possible range. ($g = 10$)

1. $R_{\max} = u^2 / g$ at $45^\circ = 900 / 10$

✓ Answer: 90 m

Q6 A stone on a 0.5 m string makes 2 revolutions per second. Find v and centripetal acceleration.

1. $\omega = 2\pi f = 4\pi \text{ rad/s}$
2. $v = r\omega = 0.5 \times 4\pi \approx 6.28 \text{ m/s}$
3. $a = \omega^2 r = 16\pi^2 \times 0.5 \approx 79 \text{ m/s}^2$

✓ Answer: $v \approx 6.28 \text{ m/s}$, $a \approx 79 \text{ m/s}^2$

Q7 A ball is thrown horizontally at 15 m/s from an 80 m cliff. Find time of flight and horizontal distance.

1. $t = \sqrt{(2h/g)} = \sqrt{16} = 4 \text{ s}$
2. $x = ut = 15 \times 4$

✓ Answer: 4 s, 60 m

Long answer questions (5 marks)

L1 Show that the path of a projectile is a parabola. Derive expressions for time of flight, maximum height and horizontal range.

Answer:

1. A body is thrown with speed u at angle θ above the horizontal. Components: $u_x = u \cos\theta$ (constant, no horizontal force) and $u_y = u \sin\theta$ (acceleration $-g$).
2. **Path:** $x = (u \cos\theta)t \Rightarrow t = x/(u \cos\theta)$. $y = (u \sin\theta)t - \frac{1}{2}gt^2 = x \tan\theta - gx^2/(2u^2 \cos^2\theta)$. This is of the form $y = bx - cx^2 \Rightarrow$ a **parabola**.
3. **Time of flight:** $y = 0$ again $\Rightarrow (u \sin\theta)T - \frac{1}{2}gT^2 = 0 \Rightarrow T = 2u \sin\theta/g$.
4. **Maximum height:** at the top $v_y = 0 \Rightarrow 0 = u^2 \sin^2\theta - 2gH \Rightarrow H = u^2 \sin^2\theta / 2g$.
5. **Range:** $R = u_x \times T = u \cos\theta \times 2u \sin\theta/g \Rightarrow R = u^2 \sin 2\theta / g$. Maximum when $\sin 2\theta = 1$, i.e. $\theta = 45^\circ$.

★ Horizontal motion is uniform, vertical motion is uniformly accelerated — together they make a parabola.

L2 Derive the expression for centripetal acceleration in uniform circular motion.

Answer:

1. A particle moves on a circle of radius r with constant speed v . At P its velocity is v_1 , after small time Δt at Q it is v_2 ; $|v_1| = |v_2| = v$ but directions differ by $\Delta\theta$.
2. Triangle of position vectors (sides $r, r, \Delta r$) and triangle of velocity vectors (sides $v, v, \Delta v$) are similar (both have angle $\Delta\theta$).
3. $\Rightarrow |\Delta v|/v = |\Delta r|/r \Rightarrow |\Delta v| = (v/r)|\Delta r|$.
4. $a = \lim |\Delta v|/\Delta t = (v/r) \lim |\Delta r|/\Delta t = (v/r) \times v \Rightarrow a_c = v^2/r = \omega^2 r$.
5. As $\Delta t \rightarrow 0$, Δv becomes perpendicular to v , i.e. it points towards the centre.

★ Even at constant speed, circular motion is accelerated because direction keeps changing.

✓ Multiple choice questions (MCQs)

1. Which is a vector?

- (a) Mass (b) Time
(c) Displacement (d) Work

2. Maximum resultant of 3 N and 4 N is

- (a) 1 N (b) 5 N
(c) 7 N (d) 12 N

3. Horizontal range is maximum when angle of projection is

- (a) 30° (b) 45°
(c) 60° (d) 90°

4. At the highest point of a projectile, velocity is

- (a) zero (b) $u \sin \theta$
(c) $u \cos \theta$ (d) u

5. $\hat{i} \times \hat{j}$ equals

- (a) 0 (b) 1
(c) $\hat{k}$ (d) $-\hat{k}$

6. Two vectors are perpendicular if their dot product is

- (a) 1 (b) 0
(c) -1 (d) maximum

7. Angles giving the same range for a given speed are

- (a) θ and 2θ (b) θ and $90^\circ - \theta$
(c) θ and $180^\circ - \theta$ (d) θ and $\theta/2$

8. Centripetal acceleration of a body moving at v in radius r is

- (a) v/r (b) v^2/r
(c) vr (d) r/v^2

Answer key: 1-(c) It has direction. | 2-(c) When parallel: $3 + 4$. | 3-(b) $\sin 2\theta = 1$. | 4-(c) Only horizontal part remains. | 5-(c) Right-hand cyclic rule. | 6-(b) $\cos 90^\circ = 0$. | 7-(b) $\sin 2\theta$ is the same. | 8-(b) $a = v^2/r$.

04 Laws of Motion

Class XI · Unit III

In simple words

- **Newton's 1st law (Inertia):** a body stays at rest or in uniform motion unless a net force acts on it.
- **2nd law:** force = rate of change of momentum, $F = dp/dt = ma$. **Impulse** = $F \times t$ = change in momentum.
- **3rd law:** every action has an equal and opposite reaction (acting on different bodies).
- **Conservation of momentum:** if no external force, total momentum stays constant (e.g. gun recoil, rocket).
- **Friction** opposes relative motion; $f = \mu N$. Static friction > kinetic friction > rolling friction. Lubrication reduces friction.
- For circular motion a **centripetal force** mv^2/r is needed. On a road, friction or banking provides it.

Remember in one line

- | | |
|--|---|
| ① A body keeps its state unless a force acts (inertia). | ② Force = rate of change of momentum ($F = ma$). |
| ③ Action and reaction are equal, opposite and act on different bodies. | ④ With no external force, total momentum stays constant. |
| ⑤ Friction opposes motion; $f = \mu N$. | ⑥ A turning vehicle needs centripetal force from friction or banking. |

Theory in detail

Aristotle vs Galileo

Aristotle believed a force is needed to keep a body moving. Galileo, using inclined planes, showed that a ball rolling down one plane rises to almost the same height on another; if the second plane is made horizontal, it would roll on forever. So a force is needed only to **change** motion, not to maintain it. Friction is the hidden force that stops things in daily life. Newton built his first law on this idea.

Free body diagram

To solve force problems: (1) isolate the body, (2) draw all forces acting on it — weight mg downward, normal reaction N perpendicular to surface, tension T along string, friction f opposite to (possible) motion, applied force, (3) choose axes, (4) apply $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$. Never include forces the body exerts on others.

Friction: friend and enemy

Friction arises from interlocking and molecular attraction between surfaces. It helps us walk, write, hold things and brake vehicles, but it wastes energy, causes wear and heating in machines.

Reducing friction: polishing, lubricants (oil, grease), ball bearings (turn sliding into rolling) and

streamlining. **Rolling friction** is much smaller than sliding friction — that is why wheels were such a great invention.

Momentum and the second law

Linear momentum $p = mv$ is the 'quantity of motion'. A heavy truck and a light car at the same speed have very different momenta, so the truck is harder to stop. The second law says force is the rate of change of momentum. It also shows that the first law is a special case: if $F = 0$, momentum and velocity stay constant.

Static and kinetic friction

When you push a heavy box gently it does not move — **static friction** exactly balances your push. As you push harder, static friction grows up to a maximum value called **limiting friction** ($\mu_s N$). Once the box starts sliding, friction drops slightly to **kinetic friction** ($\mu_k N$). That is why it is harder to start moving a box than to keep it moving.

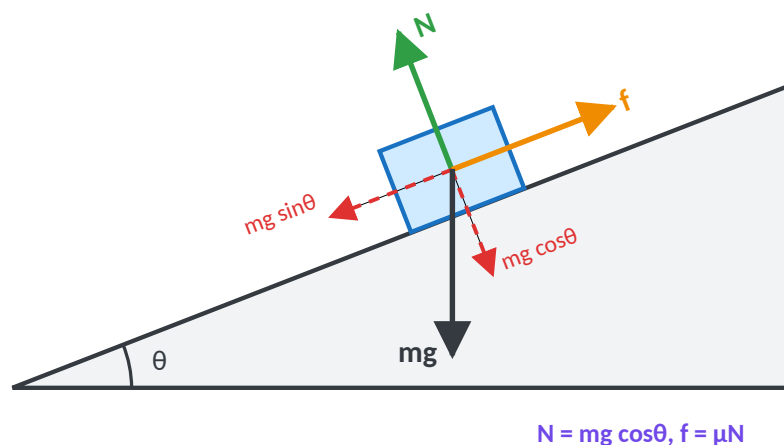


 Fig. 4.1: Free body diagram of a block on a rough inclined plane

More theory - explained

- **Types of inertia:** of rest (passengers fall back when bus starts), of motion (fall forward when it stops), of direction (thrown outward on a turn). Mass is the measure of inertia.
- **F = ma from 2nd law:** $F \propto dp/dt = m dv/dt = ma$. 1 newton is the force that gives 1 kg an acceleration of 1 m/s^2 .
- **Impulse in daily life:** a cricketer pulls hands back while catching, airbags and cushions increase contact time, so the force becomes smaller.
- **Momentum conservation proof:** in a collision $F_{12} = -F_{21}$ (3rd law) $\Rightarrow \Delta p_1 = -\Delta p_2 \Rightarrow$ total momentum unchanged.
- **Equilibrium of concurrent forces:** $\Sigma F = 0$. For three forces, **Lami's theorem:** $F_1/\sin \alpha = F_2/\sin \beta = F_3/\sin \gamma$ (angle opposite to each force).
- **Laws of friction:** limiting friction $f = \mu_s N$; independent of area of contact; depends on nature of surfaces. Angle of friction and angle of repose: $\tan \lambda = \tan \alpha = \mu_s$.

- **Apparent weight in a lift:** going up with $a \Rightarrow R = m(g + a)$; down $\Rightarrow R = m(g - a)$; free fall $\Rightarrow R = 0$ (weightless).
- **Banked road with friction:** $v_{\max} = \sqrt{[rg(\mu + \tan\theta)/(1 - \mu \tan\theta)]}$.

Key formulas

$$F = ma, p = mv$$

$$\text{Impulse } J = F\Delta t = \Delta p$$

$$\text{Level road: } v_{\max} = \sqrt{(\mu rg)}$$

$$\text{Banked road (no friction): } \tan\theta = v^2/rg$$

Quick 1-mark questions

Why do we fall forward when a bus stops suddenly?

Inertia of motion

SI unit of impulse?

N s (= kg m/s)

Which friction is self-adjusting?

Static friction

Solved problems (7)

Q1 A 20 g bullet is fired at 400 m/s from a 4 kg gun. Find the recoil speed of the gun.

1. Initial momentum = 0
2. $m_b v_b + M V = 0 \Rightarrow V = -(0.02 \times 400)/4$
3. $V = -2$ m/s (minus sign = backward)

✓ **Answer: Gun recoils at 2 m/s**

Q2 Find the maximum safe speed of a car on a level circular road of radius 20 m if $\mu = 0.5$. ($g = 10 \text{ m/s}^2$)

1. Friction gives centripetal force: $\mu mg = mv^2/r$
2. $v = \sqrt{(\mu rg)} = \sqrt{(0.5 \times 20 \times 10)} = \sqrt{100}$

✓ **Answer: $v_{\max} = 10 \text{ m/s}$ (36 km/h)**

Q3 A 60 kg man stands in a lift. Find the scale reading when lift accelerates at 2 m/s^2 (a) up (b) down. ($g = 10$)

1. (a) $R = m(g + a) = 60 \times 12 = 720 \text{ N}$
2. (b) $R = m(g - a) = 60 \times 8 = 480 \text{ N}$

✓ **Answer: 720 N; 480 N**

Q4 A 0.15 kg ball at 20 m/s is hit straight back at 20 m/s. Find impulse and average force if contact lasts 0.01 s.

1. $\Delta p = m(v - u) = 0.15 \times (20 - (-20)) = 6 \text{ N s}$
2. $F = \Delta p/\Delta t = 6/0.01$

✓ **Answer: Impulse 6 N s, $F = 600 \text{ N}$**

Q5 A 10 kg block on a rough floor ($\mu_k = 0.2$) is pushed with 50 N. Find its acceleration. ($g = 10$)

1. Friction $f = \mu mg = 0.2 \times 100 = 20$ N
2. Net force $= 50 - 20 = 30$ N $\Rightarrow a = 30/10$

✓ Answer: $a = 3 \text{ m/s}^2$

Q6 At what angle should a road of radius 40 m be banked for a speed of 20 m/s (no friction)?

1. $\tan\theta = v^2/rg = 400/(40 \times 10) = 1$

✓ Answer: $\theta = 45^\circ$

Q7 Blocks of 2 kg and 3 kg touch on a smooth floor. 10 N pushes the 2 kg block. Find acceleration and contact force.

1. $a = F/(m_1+m_2) = 10/5 = 2 \text{ m/s}^2$
2. Contact force on 3 kg $= 3 \times 2 = 6$ N

✓ Answer: $a = 2 \text{ m/s}^2$, contact force $= 6$ N



Long answer questions (5 marks)

L1 State the law of conservation of linear momentum and derive it from Newton's laws. Explain the recoil of a gun.

Answer:

1. **Statement:** if no external force acts on a system, its total linear momentum remains constant.
2. Two bodies A and B collide. During contact time Δt , A exerts F_{AB} on B and B exerts F_{BA} on A.
3. By the 2nd law: $F_{AB}\Delta t = \Delta p_B$ and $F_{BA}\Delta t = \Delta p_A$.
4. By the 3rd law: $F_{AB} = -F_{BA} \Rightarrow \Delta p_B = -\Delta p_A \Rightarrow \Delta(p_A + p_B) = 0$.
5. **Recoil of gun:** before firing total momentum $= 0$. After: $mv + MV = 0 \Rightarrow V = -mv/M$. The gun moves backward, slower than the bullet because $M > m$.

★ Momentum conservation also explains rocket propulsion and explosions.

L2 Derive the maximum safe speed of a vehicle on (a) a level circular road and (b) a banked road with friction.

Answer:

1. **(a) Level road:** centripetal force is provided only by friction. $f \leq \mu N = \mu mg$. So $mv^2/r \leq \mu mg \Rightarrow v_{\max} = \sqrt{(\mu rg)}$.
2. **(b) Banked road** at angle θ : forces are mg (down), N (perpendicular to road), f (down the slope at max speed).
3. Vertical: $N \cos\theta = mg + f \sin\theta$. Horizontal: $N \sin\theta + f \cos\theta = mv^2/r$, with $f = \mu N$.
4. Dividing: $v^2/rg = (\sin\theta + \mu \cos\theta)/(\cos\theta - \mu \sin\theta) \Rightarrow v_{\max} = \sqrt{[rg(\mu + \tan\theta)/(1 - \mu \tan\theta)]}$.

5. If $\mu = 0$: $v_o = \sqrt{rg \tan\theta}$ — optimum speed with no wear of tyres.

★ Banking lets vehicles turn safely at higher speeds.

✓ Multiple choice questions (MCQs)

1. Inertia of a body depends on its

- (a) speed (b) mass
(c) shape (d) colour

2. Newton's first law gives the idea of

- (a) energy (b) momentum
(c) inertia (d) work

3. Unit of impulse is the same as that of

- (a) force (b) momentum
(c) energy (d) power

4. Recoil of a gun is explained by

- (a) 1st law (b) conservation of momentum
(c) conservation of energy (d) law of gravitation

5. A cricketer lowers hands while catching to

- (a) increase force (b) decrease time
(c) increase time of impact (d) increase momentum

6. In a lift falling freely, apparent weight is

- (a) mg (b) $2mg$
(c) zero (d) $mg/2$

7. Coefficient of friction has

- (a) unit N (b) unit kg
(c) no unit (d) unit m/s

8. Max speed on a level curved road of radius r is

- (a) $\sqrt{\mu rg}$ (b) μrg
(c) $\sqrt{rg}$ (d) rg/μ

Answer key: 1-(b) Mass measures inertia. | 2-(c) Law of inertia. | 3-(b) Impulse = Δp . | 4-(b) Total momentum stays zero. | 5-(c) Longer time $\Rightarrow$ smaller force. | 6-(c) $R = m(g - g) = 0$. | 7-(c) Ratio of two forces. | 8-(a) $\mu mg = mv^2/r$.

05 Work, Energy and Power

Class XI · Unit IV



In simple words

- **Work** $W = F s \cos\theta$. Work is done only when the force moves the body. For a variable force, $W = \int F dx$ (area under F - x graph).
- **Kinetic energy** $K = \frac{1}{2}mv^2$. **Work-energy theorem**: net work = change in KE.
- **Potential energy**: stored energy due to position; mgh (gravity), $\frac{1}{2}kx^2$ (spring).
- **Conservative force** (gravity, spring): work does not depend on path. **Non-conservative** (friction): work depends on path.
- **Power** = work / time = $F \cdot v$. Unit watt (W). 1 hp = 746 W.
- **Elastic collision**: KE and momentum both conserved. **Inelastic**: only momentum conserved.



Remember in one line

- | | |
|--|--|
| 1 Work is done only when a force moves a body. | 2 KE is energy of motion ($\frac{1}{2}mv^2$); PE is stored energy. |
| 3 Net work done = change in KE. | 4 Conservative forces don't waste energy; friction does. |
| 5 Power is how fast work is done (watt). | 6 In elastic collisions, both KE and momentum are conserved. |



Theory in detail

What is energy?

Energy is the capacity to do work. It exists in many forms — mechanical (kinetic + potential), heat, light, sound, electrical, chemical and nuclear. Energy can change from one form to another but its total amount stays the same (**law of conservation of energy**). Einstein showed mass is also a form of energy, $E = mc^2$.

Conservative and non-conservative forces

A force is **conservative** if (i) work done depends only on start and end points, not path, (ii) work done in a closed loop is zero, and (iii) a potential energy can be defined for it. Gravity, spring force and electrostatic force are examples. **Non-conservative** forces like friction and air resistance turn mechanical energy into heat, so the work depends on path.

Power in daily life

Power tells how fast work is done. Two people climbing the same stairs do the same work, but the faster one has more power. Instantaneous power $P = F \cdot v$. Electric bills charge for energy in kWh (1 kW used for 1 hour), not for power. 1 horsepower = 746 W.

Potential energy of a spring

When a spring is stretched or compressed by x , it stores energy because work is done against the restoring force $F = -kx$. The work equals the area under the F - x graph: a triangle of base x and

height kx , giving $U = \frac{1}{2}kx^2$. Whether stretched or compressed, the stored energy is positive. Toy cars, watches and bows use this stored energy.

Motion in a vertical circle

A stone tied to a string and whirled in a vertical circle has different speeds at different points — fastest at the bottom, slowest at the top — because KE changes into PE as it rises. Tension is maximum at the bottom (must balance weight and give centripetal force) and minimum at the top. To just complete the circle the speed at the top must be at least $\sqrt{gr}$, which needs $\sqrt{5gr}$ at the bottom.

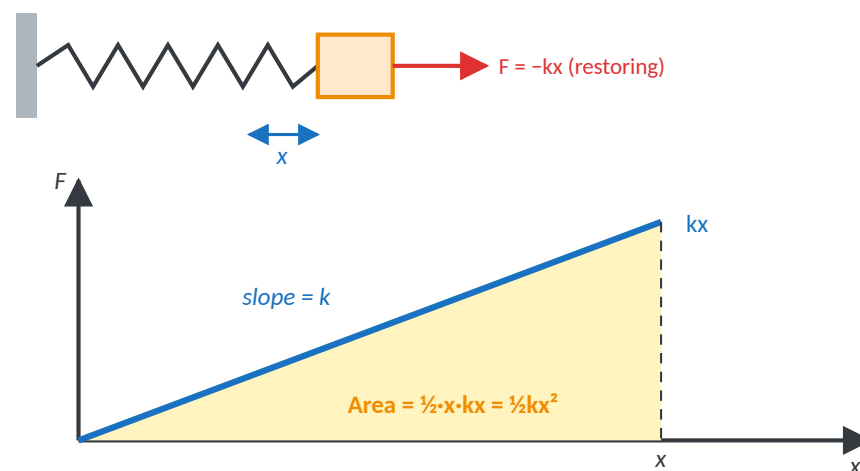


 Fig. 5.1: Spring: restoring force and the F-x graph (area = stored energy)

More theory - explained

- **Sign of work:** positive when force helps motion (lifting load up with force), negative when opposing (friction), zero when $F \perp s$ (centripetal force, a coolie walking on level ground).
- **Work-energy theorem proof:** $v^2 - u^2 = 2as \Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas = Fs = W$.
- **Variable force:** $W = \int F dx = \text{area under } F\text{-}x \text{ graph}$. For a spring ($F = -kx$) this gives $W = \frac{1}{2}kx^2$.
- **Conservation of mechanical energy:** for conservative forces $K + U$ stays constant. For a falling body, PE lost = KE gained at every point.
- **Force from PE:** $F = -dU/dx$. PE is defined only for conservative forces.
- **1-D elastic collision:** $v_1 = [(m_1 - m_2)u_1 + 2m_2u_2]/(m_1 + m_2)$, $v_2 = [(m_2 - m_1)u_2 + 2m_1u_1]/(m_1 + m_2)$. Equal masses simply **exchange velocities**.
- **Coefficient of restitution** $e = (\text{speed of separation})/(\text{speed of approach})$. $e = 1$ elastic, $0 < e < 1$ inelastic, $e = 0$ perfectly inelastic (bodies stick).
- **2-D collision:** momentum is conserved separately along x and y .
- **Vertical circle:** tension at bottom $T = mv^2/r + mg$; at top $T = mv^2/r - mg$. String just stays tight at top when $v = \sqrt{gr}$.

Key formulas

$$W = Fs \cos\theta$$

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

$$U_{\text{spring}} = \frac{1}{2}kx^2$$

$$P = W/t = Fv$$

$$\text{Vertical circle: } v_{\text{min}}(\text{top}) = \sqrt{gr}, v_{\text{min}}(\text{bottom}) = \sqrt{5gr}$$

⚡ Quick 1-mark questions

Work done by centripetal force?

Zero

1 kWh = ? J

$3.6 \times 10^6 \text{ J}$

Is momentum conserved in an inelastic collision?

Yes (KE is not)

✍ Solved problems (7)

Q1 A 10 N force pushes a 2 kg block (initially at rest) 5 m on a smooth floor. Find its final speed.

1. $W = Fs = 10 \times 5 = 50 \text{ J}$

2. By work-energy theorem, $\frac{1}{2}mv^2 = 50$

3. $v^2 = 2 \times 50 / 2 = 50 \Rightarrow v = \sqrt{50}$

✓ Answer: $v \approx 7.07 \text{ m/s}$

Q2 A motor lifts 50 kg to a height of 10 m in 5 s. Find its power. ($g = 10 \text{ m/s}^2$)

1. $\text{Work} = mgh = 50 \times 10 \times 10 = 5000 \text{ J}$

2. $\text{Power} = W/t = 5000/5$

✓ Answer: $P = 1000 \text{ W} = 1 \text{ kW}$

Q3 A 20 N force at 60° to the horizontal moves a box 5 m horizontally. Find work done.

1. $W = Fs \cos\theta = 20 \times 5 \times \cos 60^\circ$

✓ Answer: 50 J

Q4 A 1 kg ball falls from 20 m. Find its speed on reaching ground and KE at half height. ($g = 10$)

1. $v = \sqrt{2gh} = \sqrt{400} = 20 \text{ m/s}$

2. $\text{KE at } 10 \text{ m} = \text{PE lost} = mgh = 1 \times 10 \times 10$

✓ Answer: 20 m/s; 100 J

Q5 2 kg at 3 m/s hits a 1 kg ball at rest elastically. Find final velocities.

1. $v_1 = (2 - 1)/3 \times 3 = 1 \text{ m/s}$

2. $v_2 = 2 \times 2 \times 3/3 = 4 \text{ m/s}$

3. Check: momentum $6 = 2 + 4$ ✓, KE $9 = 1 + 8$ ✓

✓ Answer: 1 m/s and 4 m/s

Q6 If the balls in the last problem stick together, find the common velocity and KE lost.

1. $6 = 3v \Rightarrow v = 2 \text{ m/s}$

2. KE before = 9 J, after = $\frac{1}{2} \times 3 \times 4 = 6 \text{ J}$

✓ Answer: $v = 2 \text{ m/s}$, 3 J lost

Q7 Two bodies of 1 kg and 4 kg have equal KE. Find the ratio of their momenta.

1. $p = \sqrt{2mK} \propto \sqrt{m}$

2. $p_1/p_2 = \sqrt{(1/4)}$

✓ Answer: 1 : 2



Long answer questions (5 marks)

L1 State and prove the work-energy theorem for a variable force.

Answer:

1. **Statement:** the work done by the net force on a body equals the change in its kinetic energy.

2. $K = \frac{1}{2}mv^2 \Rightarrow dK/dt = mv(dv/dt) = (ma)v = Fv = F(dx/dt).$

3. $\Rightarrow dK = F dx$. Integrating from x_i to x_f : $K_f - K_i = \int F dx = W$.

4. So **$W = \Delta K$** for any force, constant or variable.

★ This theorem is a direct result of Newton's second law.

L2 Show that mechanical energy of a freely falling body is conserved. Also discuss elastic collision in one dimension.

Answer:

1. A body of mass m is dropped from height h (point A).

2. At A: KE = 0, PE = $mgh \Rightarrow E = mgh$.

3. At B, after falling x : $v^2 = 2gx \Rightarrow KE = mgx$, PE = $mg(h - x) \Rightarrow E = mgh$.

4. At C (ground): $v^2 = 2gh \Rightarrow KE = mgh$, PE = 0 $\Rightarrow E = mgh$. So total energy is the same everywhere.

5. **Elastic collision** (m_1, u_1 hits m_2 at rest): momentum $m_1u_1 = m_1v_1 + m_2v_2$; KE $\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$. Solving: $v_1 = (m_1 - m_2)u_1/(m_1 + m_2)$, $v_2 = 2m_1u_1/(m_1 + m_2)$.

6. Cases: $m_1 = m_2 \Rightarrow$ velocities exchange; $m_1 \ll m_2 \Rightarrow$ light body bounces back; $m_1 \gg m_2 \Rightarrow$ light body moves at $2u_1$.

★ In the absence of friction, mechanical energy is conserved.

✓ Multiple choice questions (MCQs)

1. Work done by centripetal force is

- (a) positive
- (b) negative
- (c) zero
- (d) maximum

2. If velocity of a body is doubled, its KE becomes

- (a) double
- (b) half
- (c) four times
- (d) same

3. 1 kWh equals

- (a) 3.6×10^3 J
- (b) 3.6×10^5 J
- (c) 3.6×10^6 J
- (d) 1000 J

4. Which is a conservative force?

- (a) Friction
- (b) Air drag
- (c) Gravity
- (d) Viscous force

5. PE of a spring stretched by x is

- (a) kx
- (b) $\frac{1}{2}kx$
- (c) $\frac{1}{2}kx^2$
- (d) kx^2

6. In a perfectly inelastic collision

- (a) KE conserved
- (b) bodies stick together
- (c) momentum not conserved
- (d) $e = 1$

7. SI unit of power is

- (a) joule
- (b) watt
- (c) newton
- (d) erg

8. Two equal masses collide elastically head-on (one at rest). After collision they

- (a) stick
- (b) exchange velocities
- (c) both stop
- (d) move together

Answer key: 1-(c) Force $\perp$ displacement. | 2-(c) $KE \propto v^2$. | 3-(c) $1000 \text{ W} \times 3600 \text{ s}$. | 4-(c) Work independent of path. | 5-(c) Area under F-x graph. | 6-(b) $e = 0$. | 7-(b) $1 \text{ W} = 1 \text{ J/s}$. | 8-(b) Standard result.

06 System of Particles and Rotational Motion

Class XI • Unit V

In simple words

- **Centre of mass** is the point where the whole mass of the body can be imagined to be concentrated. For a uniform rod it is at the middle.
- **Torque** ($\tau = r \times F$) is the turning effect of a force. It is the rotational version of force.
- **Moment of inertia** $I = \sum mr^2$ is the rotational version of mass. **Radius of gyration**: $I = Mk^2$.
- **Angular momentum** $L = I\omega$. If no external torque, L is conserved (a spinning dancer pulls arms in and spins faster).
- A rigid body is in **equilibrium** when net force = 0 and net torque = 0.

Remember in one line

- | | |
|---|---|
| 1 Centre of mass is the balance point of a body. | 2 Torque is the turning effect of a force ($\tau = rF \sin\theta$). |
| 3 Moment of inertia is 'rotational mass'. | 4 Angular momentum stays constant if no outside torque acts. |
| 5 Equilibrium means no net force and no net torque. | 6 Every linear quantity has a rotational partner. |

Theory in detail

Rigid body and types of motion

A **rigid body** is one whose shape and size do not change — the distance between any two particles stays fixed. It can have (i) pure **translation** (all particles have the same velocity), (ii) pure **rotation** about a fixed axis (particles move in circles), or (iii) a combination, like a rolling wheel.

Moment of inertia — rotational inertia

Just as mass resists change in linear motion, **moment of inertia** resists change in rotational motion. $I = \sum m_i r_i^2$, unit kg m^2 . The same body has different I about different axes. Mass placed far from the axis increases I a lot (r^2). A **flywheel** in engines has a heavy rim to give a large I , so its speed stays steady.

Examples of conservation of angular momentum

(1) A ballet dancer or ice skater spins faster by pulling arms in (I decreases, ω increases). (2) A diver curls up in air to make more somersaults. (3) A planet moves faster when nearer the Sun (Kepler's 2nd law). (4) A person on a rotating stool with dumbbells speeds up when the arms are folded.

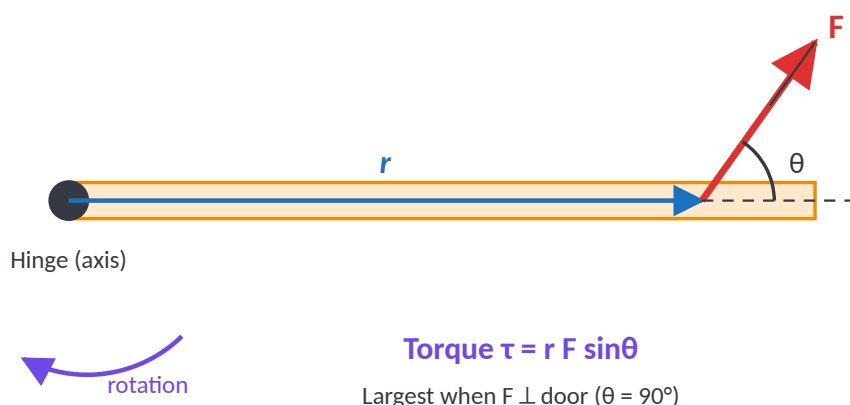
Centre of mass of a two-particle system

For two masses m_1 and m_2 on a line, the centre of mass lies between them, closer to the heavier one: $x_{\text{cm}} = (m_1 x_1 + m_2 x_2)/(m_1 + m_2)$. If the masses are equal, it lies exactly midway. The total

momentum of the system equals total mass times velocity of the centre of mass, so if no external force acts, the COM moves with constant velocity.

Radius of gyration

The **radius of gyration** k is the distance from the axis at which the whole mass could be placed to give the same moment of inertia: $I = Mk^2$. It depends on how mass is spread, not on the total mass. For a ring about its axis $k = R$, for a disc $k = R/\sqrt{2}$, and for a solid sphere $k = R\sqrt{2/5}$.



 **Fig. 6.1: Torque on a door: far from hinge + perpendicular force = maximum turning effect**

More theory - explained

- **Motion of COM:** $M a_{cm} = F_{external}$. Internal forces do not change COM motion, e.g. a shell exploding in air — its COM continues on the same parabola.
- **Torque** $\tau = rF \sin\theta$ (unit N m). A **couple** = two equal, opposite, non-collinear forces; it only rotates.
- **Angular momentum** $L = r \times p$; $\tau = dL/dt$. If $\tau = 0$, $L = \text{constant}$ (planets, divers, ballet dancers).
- **Rotational equations:** $\omega = \omega_0 + \alpha t$; $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$; $\omega^2 = \omega_0^2 + 2\alpha\theta$.
- **Linear $\leftrightarrow$ Rotational:** $x \leftrightarrow \theta$, $v \leftrightarrow \omega$, $a \leftrightarrow \alpha$, $m \leftrightarrow I$, $F \leftrightarrow \tau$, $p \leftrightarrow L$, $\frac{1}{2}mv^2 \leftrightarrow \frac{1}{2}I\omega^2$, $P = Fv \leftrightarrow P = \tau\omega$.
- **Parallel axis theorem:** $I = I_{cm} + Md^2$. **Perpendicular axis theorem** (flat bodies): $I_z = I_x + I_y$.
- **MI depends on** mass, its distribution and the axis chosen. That is why a flywheel has most mass on its rim.
- **Equilibrium of rigid body:** $\Sigma F = 0$ (no translation) and $\Sigma \tau = 0$ (no rotation). Principle of moments: clockwise moment = anticlockwise moment (lever, see-saw).
- **Rolling:** $KE = \frac{1}{2}mv^2(1 + k^2/R^2)$. The body with smaller k^2/R^2 (solid sphere) reaches the bottom first.

Key formulas

$$x_{cm} = (m_1x_1 + m_2x_2)/(m_1+m_2)$$

$$\tau = I\alpha, L = I\omega$$

Ring: MR^2 ; Disc: $\frac{1}{2}MR^2$; Solid sphere: $\frac{2}{5}MR^2$

Rod (centre): $ML^2/12$

⚡ Quick 1-mark questions

Can COM lie outside the body?

Yes, e.g. a ring

Unit of angular momentum?

$\text{kg m}^2/\text{s}$ (J s)

Why is a door handle far from hinges?

Larger r gives more torque

📝 Solved problems (7)

Q1 Masses 2 kg at $x = 0$ and 3 kg at $x = 5$ m. Find the centre of mass.

1. $x_{\text{cm}} = (2 \times 0 + 3 \times 5)/(2 + 3)$

2. $= 15/5$

✓ Answer: $x_{\text{cm}} = 3$ m (nearer the heavier mass)

Q2 A skater with $I = 4 \text{ kg m}^2$ spins at 2 rad/s. She folds her arms and I becomes 2 kg m^2 . Find new ω .

1. No external torque $\Rightarrow I_1\omega_1 = I_2\omega_2$

2. $4 \times 2 = 2 \times \omega_2$

✓ Answer: $\omega_2 = 4$ rad/s (spins twice as fast)

Q3 A 10 N force acts 0.5 m from the hinge. Find the torque when the force is (a) perpendicular (b) at 30° .

1. (a) $\tau = rF = 0.5 \times 10 = 5 \text{ N m}$

2. (b) $\tau = rF \sin 30^\circ = 5 \times 0.5$

✓ Answer: 5 N m; 2.5 N m

Q4 A wheel starts from rest with $\alpha = 2 \text{ rad/s}^2$. Find ω and angle turned in 5 s.

1. $\omega = \alpha t = 10 \text{ rad/s}$

2. $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 2 \times 25$

✓ Answer: $\omega = 10 \text{ rad/s}$, $\theta = 25 \text{ rad}$

Q5 Find the MI of a 3 kg, 2 m rod about one end using parallel axis theorem.

1. $I_{\text{cm}} = ML^2/12 = 3 \times 4/12 = 1 \text{ kg m}^2$

2. $I = I_{\text{cm}} + M(L/2)^2 = 1 + 3 \times 1$

✓ Answer: 4 kg m^2

Q6 A 30 kg child sits 2 m from the pivot. Where should a 40 kg child sit to balance?

1. $30 \times 2 = 40 \times x$

✓ Answer: $x = 1.5$ m on the other side

Q7 Find rotational KE of a body with $I = 2 \text{ kg m}^2$ spinning at 5 rad/s.

1. $K = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 2 \times 25$

✓ Answer: 25 J

Long answer questions (5 marks)

L1 Derive the relation between torque and angular momentum and state the law of conservation of angular momentum.

Answer:

- Angular momentum $L = r \times p$.
- $dL/dt = (dr/dt \times p) + (r \times dp/dt) = (v \times mv) + (r \times F)$.
- $v \times mv = 0$ (parallel vectors) $\Rightarrow dL/dt = r \times F = \tau$.
- For a rigid body rotating about a fixed axis: $L = I\omega$ and $\tau = I\alpha$.
- Conservation law:** if $\tau_{\text{ext}} = 0$, $dL/dt = 0 \Rightarrow L = I\omega = \text{constant} \Rightarrow I_1\omega_1 = I_2\omega_2$.

★ Torque is the rotational analogue of force: it changes angular momentum.

L2 Derive an expression for the kinetic energy of a rotating body and compare linear and rotational motion.

Answer:

- A rigid body rotates with angular velocity ω . A particle m_i at distance r_i has speed $v_i = r_i\omega$.
- $KE = \sum \frac{1}{2}m_i v_i^2 = \frac{1}{2}(\sum m_i r_i^2)\omega^2 = \frac{1}{2}I\omega^2$.
- Work done by torque: $W = \int \tau d\theta$; power $P = \tau\omega$.
- Comparison:** displacement $x \leftrightarrow \theta$; velocity $v \leftrightarrow \omega$; acceleration $a \leftrightarrow \alpha$; mass $m \leftrightarrow I$; force $F = ma \leftrightarrow \text{torque } \tau = I\alpha$; momentum $p = mv \leftrightarrow L = I\omega$; $KE \frac{1}{2}mv^2 \leftrightarrow \frac{1}{2}I\omega^2$; work $Fs \leftrightarrow \tau\theta$; power $Fv \leftrightarrow \tau\omega$.

★ Every linear quantity has a rotational partner.

✓ Multiple choice questions (MCQs)

1. Rotational analogue of mass is

- (a) torque (b) moment of inertia
(c) angular momentum (d) angular velocity

2. Unit of torque is

- (a) N m (b) N/m
(c) J/s (d) kg m²

3. A skater folds her arms. Her angular speed

- (a) decreases (b) increases
(c) stays same (d) becomes zero

4. MI of a ring (mass M , radius R) about its axis is

- (a) $\frac{1}{2}MR^2$ (b) MR^2
(c) $\frac{3}{5}MR^2$ (d) $2MR^2$

5. Centre of mass of a uniform rod is at

- (a) one end (b) its middle
(c) $\frac{1}{4}$ length (d) outside

6. Torque is zero when angle between r and F is

- (a) 90° (b) 45°
(c) 0° (d) 60°

7. Parallel axis theorem: $I =$

- (a) $I_{cm} - Md^2$ (b) $I_{cm} + Md^2$
(c) $I_{cm} \times Md^2$ (d) Md^2

8. Rotational KE is

- (a) $I\omega$ (b) $\frac{1}{2}I\omega^2$
(c) $\frac{1}{2}I\omega$ (d) $I^2\omega$

Answer key: 1-(b) I plays the role of m . | 2-(a) $\tau = r \times F$. | 3-(b) I decreases, L constant. | 4-(b) All mass at distance R . | 5-(b) Symmetry. | 6-(c) $\sin 0^\circ = 0$. | 7-(b) Standard theorem. | 8-(b) Analogue of $\frac{1}{2}mv^2$.

 In simple words

- **Kepler's laws:** (1) orbits are ellipses with Sun at one focus, (2) a planet sweeps equal areas in equal times, (3) $T^2 \propto r^3$.
- **Newton's law of gravitation:** every two masses attract each other, $F = Gm_1m_2/r^2$.
- **g decreases** as we go up (height) and also as we go down (depth). g is zero at the centre of Earth.
- **Escape speed:** minimum speed to leave Earth forever (≈ 11.2 km/s).
- **Orbital speed** of a satellite close to Earth ≈ 7.9 km/s.

 Remember in one line

- | | |
|---|---|
| 1 Every mass attracts every other mass. | 2 g is less at height and at depth; zero at Earth's centre. |
| 3 Planets move in ellipses; $T^2 \propto r^3$. | 4 Escape speed from Earth ≈ 11.2 km/s. |
| 5 Satellites are held in orbit by gravity. | 6 Mass stays the same everywhere; weight changes with g. |

 Theory in detail

Universal law in words

Every particle in the universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The force acts along the line joining them, is always attractive, does not depend on the medium, and forms an action–reaction pair. It is the weakest of the fundamental forces but controls stars, planets and galaxies because it has infinite range and is always attractive.

Gravitational potential energy

It is the work done in bringing a mass from infinity to a point in a gravitational field. For a mass m at distance r from Earth's centre, $U = -GMm/r$. Near the surface, the change in PE for small height h is approximately mgh — the familiar formula is a special case.

Satellites

A satellite is a body revolving around a planet. The Moon is Earth's natural satellite; artificial satellites are launched by rockets. The needed centripetal force is provided by gravity. A satellite does not fall to Earth because its horizontal speed makes it keep 'missing' the Earth while falling. India's satellites (e.g., INSAT series for communication and weather) are placed by ISRO.

Kepler's laws explained

Law of orbits: each planet moves in an ellipse with the Sun at one focus. **Law of areas:** the line joining planet and Sun sweeps equal areas in equal times, so a planet moves fastest when

nearest (perihelion) and slowest when farthest (aphelion) — this comes from conservation of angular momentum. **Law of periods:** $T^2 \propto a^3$, so outer planets take much longer to go round.

Mass vs weight

Mass is the amount of matter in a body; it is a scalar, measured in kg by a beam balance, and is the same everywhere. **Weight** is the force with which the Earth pulls it, $W = mg$; it is a vector, measured in newton by a spring balance, and changes with g . On the Moon your weight becomes about one-sixth, but your mass stays the same.

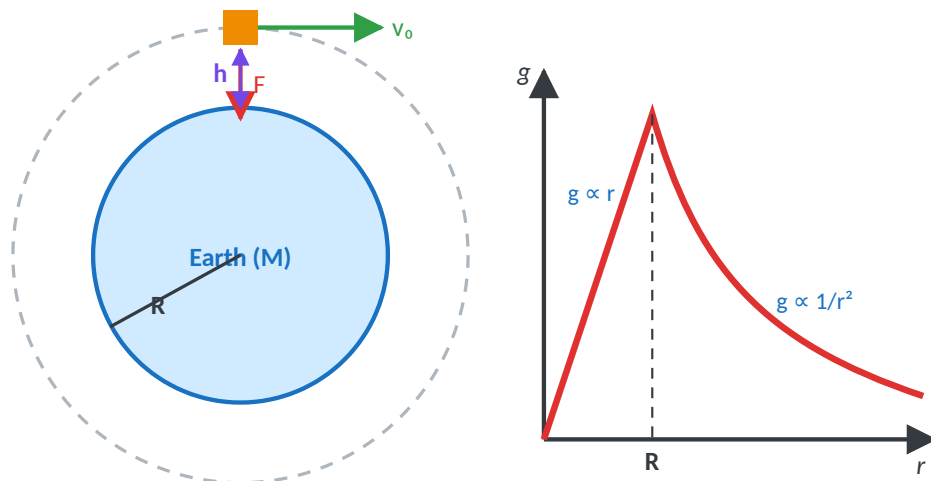


 Fig. 7.1: Satellite in orbit (left) and variation of g with distance from Earth's centre (right)

More theory - explained

- **Kepler's 3rd law from Newton:** $GMm/r^2 = mv^2/r$ with $v = 2\pi r/T \Rightarrow T^2 = (4\pi^2/GM) r^3$.
- **g and G :** $mg = GMm/R^2 \Rightarrow g = GM/R^2$. G is universal; g changes from place to place.
- **Variation of g :** decreases with height, decreases with depth (only inner mass pulls), is maximum at poles and minimum at equator (Earth's rotation and bulging).
- **Gravitational potential $V = -GM/r$; PE $U = -GMm/r$.** Negative because gravity is attractive and zero is taken at infinity.
- **Escape speed proof:** $\frac{1}{2}mv_e^2 = GMm/R \Rightarrow v_e = \sqrt{2GM/R} = \sqrt{2gR}$. It does not depend on the mass of the body.
- **Satellite:** $v_0 = \sqrt{GM/r}$, $T = 2\pi\sqrt{r^3/GM}$. Near Earth, $v_e = \sqrt{2} v_0$. Energy: $KE = GMm/2r$, $PE = -GMm/r$, total = $-GMm/2r$.
- **Geostationary satellite:** $T = 24$ h, height $\approx 36\,000$ km, moves west to east over the equator; used for communication. **Polar satellites:** low height, used for weather and remote sensing.
- **Weightlessness** in a satellite: astronaut and satellite fall freely together, so normal reaction is zero.

Key formulas

$$F = Gm_1m_2/r^2, \quad G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$$

$$g = GM/R^2$$

$$g_h = g(1 + h/R)^{-2} \approx g(1 - 2h/R); g_d = g(1 - d/R)$$

$$v_e = \sqrt{2gR}, v_o = \sqrt{gR}$$

$$U = -GMm/r$$

✚ Quick 1-mark questions

Value of G on the Moon?

Same, $6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$

Where is g maximum on Earth?

At the poles

Why is total energy of a satellite negative?

It is bound to Earth

✎ Solved problems (7)

Q1 Find the escape speed from Earth. ($g = 9.8 \text{ m/s}^2$, $R = 6.4 \times 10^6 \text{ m}$)

$$1. v_e = \sqrt{2gR} = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$2. = \sqrt{1.254 \times 10^8} \approx 1.12 \times 10^4 \text{ m/s}$$

✓ Answer: $v_e \approx 11.2 \text{ km/s}$

Q2 Find g at (a) height $h = R$ and (b) depth $d = R/2$. Take $g = 9.8 \text{ m/s}^2$ on surface.

$$1. (a) g_h = g/(1 + h/R)^2 = g/(2)^2 = g/4$$

$$2. (b) g_d = g(1 - d/R) = g(1 - 1/2) = g/2$$

✓ Answer: (a) 2.45 m/s^2 (b) 4.9 m/s^2

Q3 Find the gravitational force between two 100 kg masses 1 m apart.

$$1. F = Gm_1m_2/r^2 = 6.67 \times 10^{-11} \times 10^4/1$$

✓ Answer: $6.67 \times 10^{-7} \text{ N}$ (very small)

Q4 Find the orbital speed of a satellite close to Earth's surface. ($g = 9.8$, $R = 6.4 \times 10^6 \text{ m}$)

$$1. v_o = \sqrt{gR} = \sqrt{6.27 \times 10^7}$$

✓ Answer: $\approx 7.9 \text{ km/s}$

Q5 If Earth shrinks to half its radius with the same mass, what happens to g?

$$1. g = GM/R^2 \Rightarrow R \rightarrow R/2 \text{ gives } g \rightarrow 4g$$

✓ Answer: g becomes 4 times

Q6 A planet's orbit radius is 4 times Earth's. Find its year.

$$1. T^2 \propto r^3 \Rightarrow T = (4)^{3/2} \times 1 \text{ year} = 8$$

✓ Answer: 8 years

Q7 A 60 kg man goes to the Moon ($g_{\text{moon}} = g/6$). Find his mass and weight there. ($g = 9.8$)

1. Mass stays the same = 60 kg
2. Weight = $60 \times 9.8/6$

✓ Answer: 60 kg; 98 N

Long answer questions (5 marks)

L1 Derive expressions for the variation of acceleration due to gravity with (a) altitude and (b) depth.

Answer:

1. On the surface: $g = GM/R^2$.
2. **(a) Height h:** $g_h = GM/(R+h)^2 \Rightarrow g_h/g = R^2/(R+h)^2 = (1+h/R)^{-2}$.
3. For $h \ll R$ (binomial): $g_h = g(1 - 2h/R)$. g decreases with height.
4. **(b) Depth d:** only the inner sphere of radius $(R - d)$ attracts. Its mass $M' = M(R - d)^3/R^3$ (uniform density).
5. $g_d = GM'/(R - d)^2 = GM(R - d)/R^3 \Rightarrow g_d = g(1 - d/R)$. At the centre ($d = R$), $g = 0$.

★ g is maximum at the surface and decreases both upward and downward.

L2 Derive the expressions for escape speed and orbital speed. Show that $v_e = \sqrt{2} v_o$.

Answer:

1. **Escape speed:** work needed to take mass m from the surface to infinity = $\int_R^\infty GMm/r^2 dr = GMm/R$.
2. This must equal the KE given: $\frac{1}{2}mv_e^2 = GMm/R \Rightarrow v_e = \sqrt{(2GM/R)} = \sqrt{(2gR)} \approx 11.2 \text{ km/s}$.
3. **Orbital speed** at radius r : $mv_o^2/r = GMm/r^2 \Rightarrow v_o = \sqrt{(GM/r)}$.
4. Near the surface ($r \approx R$): $v_o = \sqrt{(gR)} \approx 7.9 \text{ km/s}$.
5. Therefore $v_e = \sqrt{2} v_o$. Period $T = 2\pi r/v_o = 2\pi\sqrt{(r^3/GM)}$.

★ Increasing a satellite's speed by about 41% would let it escape from Earth.

✓ Multiple choice questions (MCQs)

1. Value of G is

- (a) 9.8 N/kg (b) $6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$
(c) 6.67×10^{11} (d) same as g

2. Acceleration due to gravity at Earth's centre is

- (a) g (b) $2g$
(c) zero (d) infinite

3. Escape speed from Earth is about

- (a) 7.9 km/s (b) 11.2 km/s
(c) 3 km/s (d) 30 km/s

4. Kepler's law of areas follows from conservation of

- (a) energy (b) linear momentum
(c) angular momentum (d) mass

5. Period of a geostationary satellite is

- (a) 12 h (b) 24 h
(c) 1 h (d) 365 days

6. g is maximum at

- (a) equator (b) poles
(c) centre (d) mountain top

7. Escape speed does NOT depend on

- (a) mass of planet (b) radius of planet
(c) mass of the body (d) g

8. If distance between two masses is doubled, force becomes

- (a) half (b) double
(c) one-fourth (d) four times

Answer key: 1-(b) Universal constant. | 2-(c) $g_d = g(1 - d/R)$, $d = R$. | 3-(b) $\sqrt{2gR}$. | 4-(c) No torque on planet. | 5-(b) Same as Earth's rotation. | 6-(b) Smallest radius, no rotation effect. | 7-(c) $v_e = \sqrt{2GM/R}$. | 8-(c) $F \propto 1/r^2$.

08 Mechanical Properties of Solids

Class XI • Unit VII

In simple words

- **Elasticity**: property of a body to regain its shape after the force is removed.
- **Stress** = restoring force per unit area (N/m^2). **Strain** = change in size / original size (no unit).
- **Hooke's law**: within elastic limit, stress $\propto$ strain. Constant = modulus of elasticity.
- Three moduli: **Young's** (length), **Bulk** (volume), **Shear / Rigidity** (shape).
- **Poisson's ratio** = lateral strain / longitudinal strain.
- Stress-strain graph: proportional limit $\rightarrow$ elastic limit $\rightarrow$ yield point $\rightarrow$ fracture point.

Remember in one line

- | | |
|--|---|
| 1 Elasticity is the ability to regain shape. | 2 Stress = force/area; strain = change/original. |
| 3 Within elastic limit, stress $\propto$ strain (Hooke's law). | 4 Young's modulus for length, bulk for volume, rigidity for shape. |
| 5 Steel is more elastic than rubber. | 6 Stretched wires store energy = $\frac{1}{2} \times \text{load} \times \text{extension}$. |

Theory in detail

Hooke's law in simple words

Pull a spring gently and it stretches a little; pull twice as hard and it stretches twice as much. This proportional behaviour holds only up to the **elastic limit**. Beyond it the material gets permanently deformed. So, within the elastic limit, **stress $\propto$ strain**, and the ratio is called the **modulus of elasticity** (unit N/m^2).

Three moduli explained

Young's modulus (Y) — for change in length of wires/rods; found only for solids. **Bulk modulus (B)** — for change in volume under uniform pressure; found for solids, liquids and gases. **Modulus of rigidity (η)** — for change in shape when a tangential force acts; found only for solids (liquids cannot resist shear). Steel has large Y, so it is more elastic than rubber.

Elastic fatigue and elastic after-effect

If a body is stressed again and again, it loses strength — **elastic fatigue** (why bridges are declared unsafe after long use). Some materials take time to regain shape after the force is removed — **elastic after-effect** (quartz shows very little, so it is used in galvanometer suspensions).

Hydraulic stress and bulk modulus

When a body is placed in a fluid under high pressure, it is squeezed equally from all sides; its shape stays the same but its volume decreases. The restoring force per unit area here equals the applied pressure, called hydraulic stress. $B = -P/(\Delta V/V)$; the minus sign shows that volume decreases as pressure increases. Steel's B is about 160 GPa, water's about 2.2 GPa, air's about 10^{-4} GPa.

Poisson's ratio in practice

When a rubber band is stretched, it becomes longer and thinner at the same time. The ratio of the fractional decrease in width to the fractional increase in length is **Poisson's ratio**. For most metals it is about 0.3, for rubber nearly 0.5 (volume almost unchanged). It is a pure number with no unit.

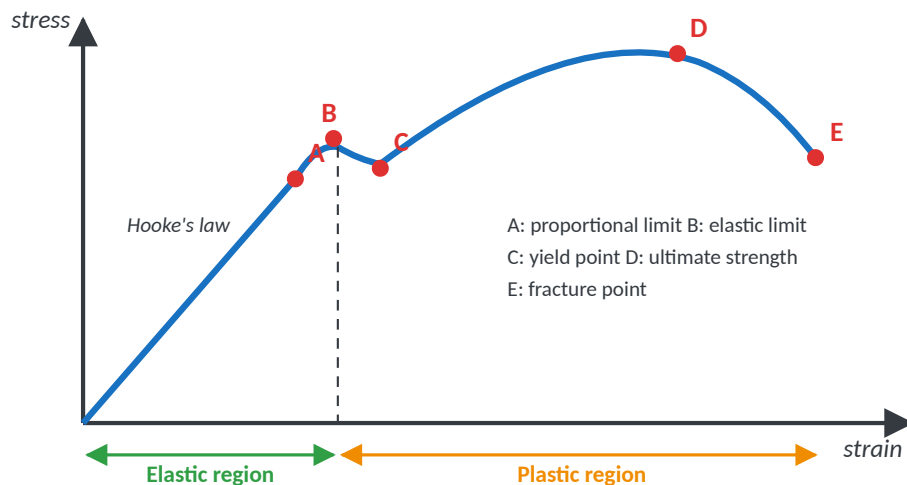


 Fig. 8.1: Stress-strain curve for a metal wire

More theory - explained

- **Origin of elasticity:** interatomic forces pull atoms back to their normal positions. Perfectly elastic (quartz $\approx$) and plastic (putty) are the two extremes.
- **Types of stress:** tensile / compressive (change length), tangential or shearing (change shape), hydraulic (change volume).
- **Stress-strain curve:** O $\rightarrow$ A straight (Hooke's law); B yield point; beyond it permanent set; D = ultimate tensile strength; E = fracture. Long plastic region $\Rightarrow$ **ductile** (copper); little $\Rightarrow$ **brittle** (glass).
- **Elastomers** like rubber and aorta stretch a lot, have no straight-line region and regain shape.
- **Compressibility** $= 1/B$. Gases are most compressible, solids least.
- **Shear modulus** $\eta = F/(A\theta)$, usually $\eta \approx Y/3$. **Poisson's ratio** $\sigma = (\Delta d/d)/(\Delta L/L)$, no unit, lies between -1 and 0.5 .
- **Energy density** (energy per unit volume) $= \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{1}{2}Y(\text{strain})^2$.
- **Applications:** I-shaped girders (strong, light), thickness of crane ropes chosen by elastic limit, maximum height of mountains (~ 10 km) limited by rock's elastic limit.

Key formulas

$$Y = (F/A)/(\Delta L/L) = FL/(A\Delta L)$$

$$B = -\Delta P/(\Delta V/V)$$

$$\eta = \text{shear stress} / \text{shear strain}$$

$$\text{Elastic PE} = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} = \frac{1}{2}F\Delta L$$

➤ Quick 1-mark questions

More elastic: rubber or steel?

Steel

Unit of strain?

No unit

Why are girders I-shaped?

Less weight, same strength against bending

📎 Solved problems (6)

Q1 A wire 2 m long, area 1 mm^2 , stretches 1 mm under 100 N. Find Young's modulus.

1. $A = 1 \text{ mm}^2 = 10^{-6} \text{ m}^2$, $\Delta L = 10^{-3} \text{ m}$

2. $Y = FL/(\Delta L A) = (100 \times 2)/(10^{-6} \times 10^{-3})$

✓ Answer: $Y = 2 \times 10^{11} \text{ N/m}^2$

Q2 In the above wire, find the elastic energy stored.

1. $U = \frac{1}{2} F \Delta L = \frac{1}{2} \times 100 \times 10^{-3}$

✓ Answer: $U = 0.05 \text{ J}$

Q3 A steel wire ($L = 1 \text{ m}$, $r = 0.5 \text{ mm}$) carries 10 kg. Find its extension. ($Y = 2 \times 10^{11} \text{ N/m}^2$, $g = 10$)

1. $A = \pi r^2 = 3.14 \times (5 \times 10^{-4})^2 = 7.85 \times 10^{-7} \text{ m}^2$

2. $\Delta L = FL/(AY) = 100 \times 1/(7.85 \times 10^{-7} \times 2 \times 10^{11})$

✓ Answer: $\Delta L \approx 6.4 \times 10^{-4} \text{ m} \approx 0.64 \text{ mm}$

Q4 Water ($B = 2.2 \times 10^9 \text{ Pa}$) is put under an extra pressure of $1.1 \times 10^7 \text{ Pa}$. Find the fractional change in volume.

1. $\Delta V/V = \Delta P/B = 1.1 \times 10^7/2.2 \times 10^9$

✓ Answer: 5×10^{-3} (0.5%)

Q5 A wire's length increases by 0.2% and its diameter decreases by 0.06%. Find Poisson's ratio.

1. $\sigma = \text{lateral strain} / \text{longitudinal strain} = 0.06/0.2$

✓ Answer: 0.3

Q6 Stress in a wire is $2 \times 10^8 \text{ N/m}^2$, $Y = 2 \times 10^{11}$. Find energy per unit volume.

1. $\text{strain} = \text{stress}/Y = 10^{-3}$

2. $u = \frac{1}{2} \times 2 \times 10^8 \times 10^{-3}$

✓ Answer: 10^5 J/m^3

Long answer questions (5 marks)

L1 Draw and explain the stress-strain curve for a metal wire.

Answer:

1. A wire is loaded gradually and stress is plotted against strain.
2. **O to A** (proportional limit): straight line; Hooke's law obeyed; slope = Y .
3. **A to B** (elastic limit/yield point): not straight but the wire still returns to original length on unloading.
4. **B to D** (plastic region): strain increases rapidly; on unloading there is a permanent set. D is the **ultimate tensile strength** (maximum stress).
5. **D to E**: the wire thins at one place (necking) and breaks at E, the **fracture point**.
6. If D and E are far apart the material is **ductile** (copper, aluminium); if close, it is **brittle** (cast iron, glass).

★ The curve tells how strong and how ductile a material is.

L2 Derive the expression for elastic potential energy stored in a stretched wire.

Answer:

1. Wire of length L and area A is stretched by a force. At extension x , restoring force $F = YAx/L$.
2. Work to stretch further by dx : $dW = F dx = (YA/L)x dx$.
3. Total work for extension l : $W = \int_0^l (YA/L)x dx = YAl^2/2L$.
4. Rewrite: $W = \frac{1}{2} \times (YAl/L) \times l = \frac{1}{2} \times \text{load} \times \text{extension}$.
5. Energy per unit volume $u = W/(AL) = \frac{1}{2}(Yl/L)(l/L) = \frac{1}{2} \times \text{stress} \times \text{strain}$.

★ This work is stored as elastic potential energy in the wire.

Multiple choice questions (MCQs)

1. SI unit of stress is

- (a) N (b) N/m^2
(c) N m (d) no unit

2. Hooke's law is valid

- (a) always (b) within elastic limit
(c) beyond yield point (d) only for liquids

3. Which is more elastic?

- (a) Rubber (b) Steel
(c) Clay (d) Putty

4. Modulus of rigidity is related to change in

- (a) length (b) volume
(c) shape (d) mass

5. Poisson's ratio has

- (a) unit N/m^2 (b) no unit
(c) unit m (d) unit N

6. Bulk modulus exists for

- (a) solids only (b) liquids only
(c) gases only (d) solids, liquids and gases

7. Energy stored per unit volume in a stretched wire is

- | | |
|------------------------------|--|
| (a) stress $\times$ strain | (b) $\frac{1}{2}$ stress $\times$ strain |
| (c) 2 stress $\times$ strain | (d) stress/strain |

8. Materials that break soon after the elastic limit are

- | | |
|-------------|----------------|
| (a) ductile | (b) brittle |
| (c) plastic | (d) elastomers |

Answer key: 1-(b) Force per area. | 2-(b) Linear region only. | 3-(b) Higher Y. | 4-(c) Shear. | 5-(b) Ratio of strains. | 6-(d) All can be compressed. | 7-(b) Standard result. | 8-(b) Glass, cast iron.

09 Mechanical Properties of Fluids

Class XI • Unit VII

In simple words

- **Pressure** in a liquid increases with depth: $P = P_0 + \rho gh$.
- **Pascal's law**: pressure applied to an enclosed liquid is passed equally in all directions (hydraulic lift, brakes).
- **Viscosity** is liquid friction. A falling ball reaches a constant **terminal velocity** (Stokes' law $F = 6\pi\eta rv$).
- **Bernoulli's theorem**: along a streamline, $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$. Faster flow $\Rightarrow$ lower pressure (aeroplane wing).
- **Surface tension**: liquid surface acts like a stretched skin. It causes drops to be spherical and capillary rise.

Remember in one line

- | | |
|---|--|
| 1 Pressure in a liquid increases with depth ($P = P_0 + \rho gh$). | 2 Pascal: pressure applied is passed equally everywhere. |
| 3 Narrow pipe $\Rightarrow$ faster flow ($A_1v_1 = A_2v_2$). | 4 Faster flow $\Rightarrow$ lower pressure (Bernoulli). |
| 5 Viscosity is liquid friction; falling bodies reach terminal velocity. | 6 Surface tension makes liquid surfaces act like stretched skin. |

Theory in detail

What is a fluid?

Liquids and gases can flow, so they are called **fluids**. They have no fixed shape and cannot resist a shear force for long. A fluid at rest exerts force only perpendicular to any surface — that is why pressure is defined as normal force per unit area. Pressure at a point acts equally in all directions.

Surface energy

Molecules inside a liquid are pulled equally from all sides, but a molecule at the surface is pulled only inward. So surface molecules have extra energy, called **surface energy**. To increase the surface area, work must be done. Surface tension T equals surface energy per unit area. This is why liquids try to reduce their surface area.

Angle of contact

The angle between the tangent to the liquid surface and the solid surface inside the liquid is the **angle of contact** θ . If adhesion (liquid–solid attraction) is stronger than cohesion, $\theta < 90^\circ$ and the liquid wets the surface (water on clean glass). If cohesion is stronger, $\theta > 90^\circ$ and it does not wet (mercury on glass). Waterproofing agents increase θ ; detergents decrease it.

Effect of gravity on fluid pressure

In a fluid at rest, pressure increases with depth because deeper layers support the weight of the fluid above them: $P = P_a + \rho gh$. Points at the same depth in the same liquid have the same

pressure, regardless of the vessel's shape. Dams are made thicker at the bottom; divers feel ear pain deep under water; water supply tanks are placed high to get good pressure.

Drops and bubbles

Because of surface tension, pressure inside a curved liquid surface is more than outside. For a liquid drop (one surface) the excess pressure is $2T/r$; for a soap bubble (two surfaces) it is $4T/r$. Smaller drops and bubbles have greater excess pressure. If two bubbles of different sizes are connected, air flows from the smaller to the larger one.

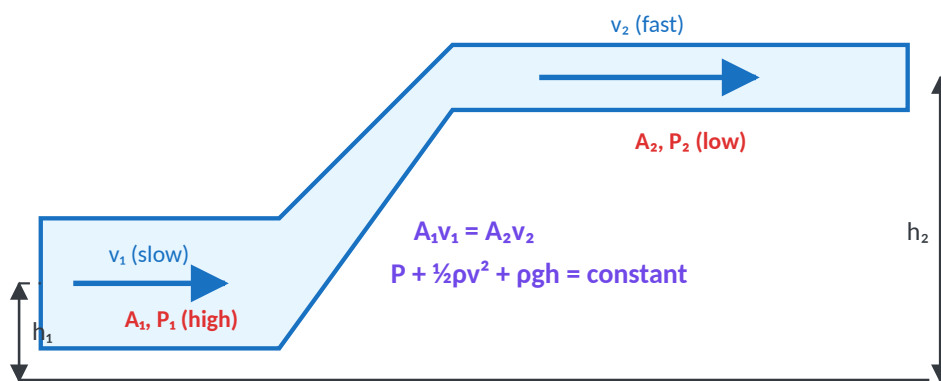


Fig. 9.1: Bernoulli's theorem: where the pipe narrows, speed rises and pressure falls

More theory - explained

- **Pressure facts:** scalar, unit pascal; $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa} = 76 \text{ cm of Hg}$. **Gauge pressure** = $P - P_a = \rho gh$. Pressure depends only on depth, not on the shape of the vessel (hydrostatic paradox).
- **Hydraulic machines:** small force on small piston gives large force on big piston, $F_2 = F_1 A_2 / A_1$ (lifts, brakes). Energy is still conserved: small piston moves more.
- **Equation of continuity:** $A_1 v_1 = A_2 v_2$ — water flows faster where the pipe (or river) is narrow.
- **Streamline and turbulent flow:** Reynolds number $R_e = \rho v d / \eta$. $R_e < 1000$ streamline, > 2000 turbulent. Critical velocity $v_c = R_e \eta / \rho d$.
- **Bernoulli applications:** speed of efflux $v = \sqrt{2gh}$ (Torricelli); Venturimeter; atomiser/sprayer; lift on aeroplane wings; roofs blown off in storms; spinning ball swings (Magnus effect).
- **Viscosity:** $F = -\eta A (dv/dx)$; unit Pa s ($1 \text{ Pa s} = 10 \text{ poise}$). Viscosity of liquids **decreases** with temperature; of gases **increases**.
- **Terminal velocity:** when weight = buoyancy + viscous drag, net force is zero and the body falls at constant speed.
- **Surface tension** $T = F/L$ = surface energy per unit area. Acute angle of contact (water-glass) $\Rightarrow$ liquid rises in capillary; obtuse (mercury-glass) $\Rightarrow$ it falls. Soaps and detergents lower T ; T decreases with temperature.

Key formulas

$$P = P_0 + \rho gh$$

$$F_1/A_1 = F_2/A_2$$

$$v_t = 2r^2(\rho - \sigma)g / 9\eta$$

Excess pressure: drop $2T/r$, bubble $4T/r$

$$\text{Capillary rise } h = 2T \cos\theta / \rho g$$

Quick 1-mark questions

Why are small drops spherical?

Surface tension makes surface area minimum

Effect of temperature on viscosity of liquids?

Decreases

Why is blood pressure higher at feet?

Greater depth h , so ρgh is larger

Solved problems (9)

Q1 Find total pressure 10 m below the water surface. ($P_0 = 1.01 \times 10^5$ Pa, $\rho = 1000$ kg/m³, $g = 10$)

1. $P = P_0 + \rho gh = 1.01 \times 10^5 + 1000 \times 10 \times 10$

2. $= 1.01 \times 10^5 + 1.0 \times 10^5$

✓ Answer: $P \approx 2.01 \times 10^5$ Pa (about 2 atm)

Q2 A hydraulic lift has pistons of area 0.01 m² and 0.5 m². What load can 100 N on the small piston lift?

1. By Pascal's law $F_2 = F_1 (A_2/A_1)$

2. $= 100 \times 0.5/0.01 = 100 \times 50$

✓ Answer: $F_2 = 5000$ N

Q3 Find the excess pressure inside a soap bubble of radius 1 cm. ($T = 0.03$ N/m)

1. Bubble has 2 surfaces: $\Delta P = 4T/r$

2. $= 4 \times 0.03 / 0.01$

✓ Answer: $\Delta P = 12$ Pa

Q4 Water flows at 2 m/s in a pipe of area 4 cm² that narrows to 1 cm². Find speed in the narrow part.

1. $A_1 v_1 = A_2 v_2 \Rightarrow v_2 = 4 \times 2/1$

✓ Answer: 8 m/s

Q5 In the above horizontal pipe, $P_1 = 2 \times 10^5$ Pa. Find P_2 . ($\rho = 1000 \text{ kg/m}^3$)

1. $P_2 = P_1 - \frac{1}{2}\rho(v_2^2 - v_1^2) = 2 \times 10^5 - 500 \times (64 - 4)$

2. $= 2 \times 10^5 - 3 \times 10^4$

✓ Answer: 1.7×10^5 Pa

Q6 Water leaks from a hole 5 m below the water level of a tank. Find the speed of efflux. ($g = 10$)

1. $v = \sqrt{2gh} = \sqrt{2 \times 10 \times 5}$

✓ Answer: 10 m/s

Q7 Find terminal velocity of a rain drop of radius 0.1 mm in air ($\eta = 1.8 \times 10^{-5} \text{ Pa s}$, ignore air density).

1. $v = \frac{2r^2\rho g}{9\eta} = \frac{2 \times 10^{-8} \times 1000 \times 9.8}{9 \times 1.8 \times 10^{-5}}$

2. $= 1.96 \times 10^{-4} / 1.62 \times 10^{-4}$

✓ Answer: $\approx 1.2 \text{ m/s}$

Q8 Find the rise of water in a capillary of radius 0.5 mm. ($T = 0.07 \text{ N/m}$, $\theta = 0^\circ$, $g = 10$)

1. $h = \frac{2T \cos\theta}{r\rho g} = \frac{2 \times 0.07}{(5 \times 10^{-4} \times 1000 \times 10)}$

✓ Answer: $h = 0.028 \text{ m} = 2.8 \text{ cm}$

Q9 Find work done to blow a soap bubble of radius 1 cm. ($T = 0.03 \text{ N/m}$)

1. Two surfaces: area increase $= 2 \times 4\pi r^2 = 8\pi \times 10^{-4} \text{ m}^2$

2. $W = T \times \Delta A = 0.03 \times 8\pi \times 10^{-4}$

✓ Answer: $\approx 7.5 \times 10^{-5} \text{ J}$



Long answer questions (5 marks)

L1 State and prove Bernoulli's theorem. Give two applications.

Answer:

- Statement:** for streamline flow of an ideal (non-viscous, incompressible) fluid, $P + \frac{1}{2}\rho v^2 + \rho gh$ is constant along a streamline.
- Consider fluid in a pipe from section 1 (A_1, v_1, P_1, h_1) to section 2 (A_2, v_2, P_2, h_2). In time Δt , volume $\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t$ passes.
- Work done by pressure $= P_1 \Delta V - P_2 \Delta V$.
- Change in KE $= \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2)$; change in PE $= \rho \Delta V g (h_2 - h_1)$.
- Work-energy theorem: $(P_1 - P_2) \Delta V = \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2) + \rho g \Delta V (h_2 - h_1) \Rightarrow P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$.

6. **Applications:** (1) Dynamic lift on aircraft wings — air moves faster above, pressure is lower, so net upward force. (2) Torricelli's law $v = \sqrt{2gh}$ for speed of efflux.

★ **Bernoulli's theorem is the law of conservation of energy for flowing fluids.**

L2 What is terminal velocity? Derive its expression using Stokes' law.

Answer:

1. When a small sphere falls through a viscous fluid, three forces act: weight (down), buoyancy (up) and viscous drag $F = 6\pi\eta r v$ (up), which grows with speed.
2. Soon the upward forces balance the weight; net force becomes zero and the sphere moves with constant **terminal velocity**.
3. Weight = $(4/3)\pi r^3 \rho g$; buoyancy = $(4/3)\pi r^3 \sigma g$ (σ = fluid density).
4. $(4/3)\pi r^3 (\rho - \sigma) g = 6\pi\eta r v_t \Rightarrow v_t = 2r^2(\rho - \sigma)g/9\eta$.
5. So $v_t \propto r^2$: big raindrops fall faster; tiny cloud droplets float. If $\sigma > \rho$ (air bubble in water), v_t is negative — it rises.

★ **Terminal velocity explains why raindrops don't hurt us and why parachutes work.**

✓ Multiple choice questions (MCQs)

1. Hydraulic lift works on

- (a) Archimedes' principle
(b) Pascal's law
(c) Bernoulli's theorem
(d) Stokes' law

2. Pressure at depth h in a liquid is

- (a) ρgh
(b) $\rho/g h$
(c) gh/ρ
(d) ph/g

3. Water flows faster where a pipe is

- (a) wider
(b) narrower
(c) higher
(d) lower

4. Lift on an aeroplane wing is explained by

- (a) Pascal's law
(b) Bernoulli's theorem
(c) Hooke's law
(d) Newton's 1st law

5. Terminal velocity of a sphere is proportional to

- (a) r
(b) r^2
(c) $1/r$
(d) r^3

6. Excess pressure inside a soap bubble is

- (a) $2T/r$
(b) $4T/r$
(c) T/r
(d) $8T/r$

7. Surface tension of a liquid on heating

- (a) increases
(b) decreases
(c) stays same
(d) becomes infinite

8. Mercury in a glass capillary

- (a) rises
(b) falls
(c) stays level
(d) boils

Answer key: 1-(b) Pressure transmitted equally. | 2-(a) Gauge pressure. | 3-(b) Continuity. | 4-(b) Faster air above $\Rightarrow$ lower pressure. | 5-(b) $v_t = 2r^2(\rho - \sigma)g/9\eta$. | 6-(b) Two surfaces. | 7-(b) Molecular forces weaken. | 8-(b) Angle of contact $> 90^\circ$.

10 Thermal Properties of Matter

Class XI • Unit VII

In simple words

- **Heat** is energy that flows due to temperature difference. **Temperature** tells how hot a body is.
- Solids, liquids and gases **expand** on heating. Water is special: it contracts from 0°C to 4°C (**anomalous expansion**), so it is densest at 4°C.
- **Specific heat** c : heat needed to raise 1 kg by 1 K. For gases, $C_p > C_v$.
- **Latent heat**: heat used to change state without changing temperature.
- Heat moves by **conduction** (solids), **convection** (fluids) and **radiation** (no medium needed).
- **Wien's law**: hotter body glows at shorter wavelength. **Stefan's law**: power radiated $\propto T^4$.

Remember in one line

- | | |
|--|---|
| 1 Heat is energy that flows because of temperature difference. | 2 Solids, liquids and gases expand on heating. |
| 3 Water is densest at 4 °C. | 4 Latent heat changes state without changing temperature. |
| 5 Heat travels by conduction, convection and radiation. | 6 Hotter bodies glow bluer and radiate much more ($\propto T^4$). |

Theory in detail

Heat vs temperature

Heat is energy in transit due to temperature difference; unit joule (or calorie, 1 cal = 4.186 J). **Temperature** is a measure of hotness and decides the direction of heat flow — heat always flows from higher to lower temperature. A bucket of warm water may have more heat than a cup of boiling water, but the cup has higher temperature.

Specific heat of gases: C_p and C_v

A gas can be heated at constant volume or at constant pressure. At constant volume all heat raises internal energy. At constant pressure, the gas also expands and does work on the surroundings, so extra heat is needed. Hence $C_p > C_v$, and for an ideal gas $C_p - C_v = R$.

Blackbody radiation in daily life

A perfect black body absorbs all radiation falling on it and emits the maximum possible radiation at every temperature. As temperature increases, a hot iron glows dull red $\rightarrow$ orange $\rightarrow$ yellow $\rightarrow$ white, because the peak wavelength shifts to shorter values (Wien's law) and total emission rises sharply (Stefan's law). Light-coloured clothes are worn in summer since they absorb less radiation.

Anomalous expansion of water

Most liquids expand when heated, but water between 0 °C and 4 °C contracts on heating and expands on cooling. So water has **maximum density at 4 °C**. In winter, the surface water of a lake

cools to 4 °C and sinks; below 4 °C it stays on top and freezes. Ice, being a poor conductor, insulates the water below, allowing fish to survive.

Calorimetry and latent heat

A **calorimeter** is an insulated copper vessel used to measure heat. In the **method of mixtures**, heat lost by a hot body = heat gained by cold water and the calorimeter. During a change of state (melting, boiling), heat supplied does not raise temperature but is used to break molecular bonds; this is **latent heat**. That is why steam at 100 °C causes worse burns than water at 100 °C.

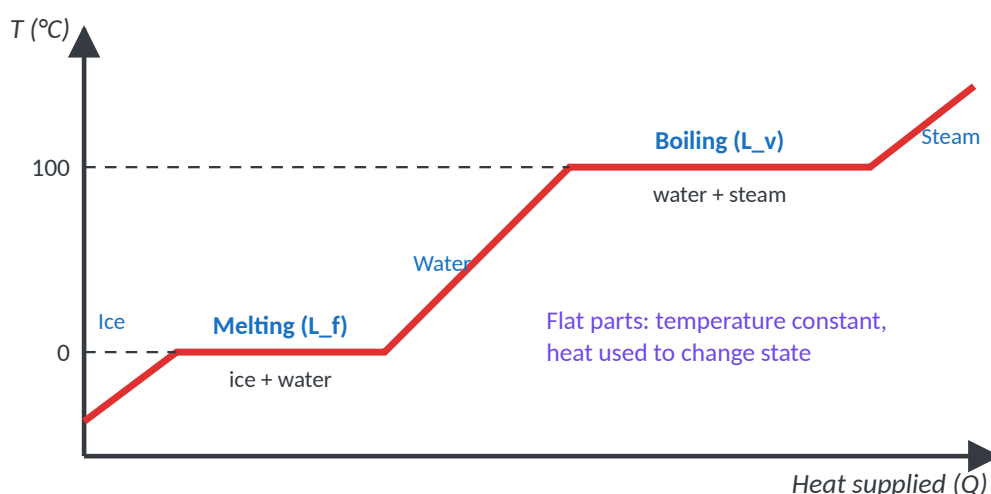


Fig. 10.1: Heating curve of water: latent heat is absorbed at the flat parts

More theory - explained

- **Temperature scales:** $C/100 = (F - 32)/180 = (K - 273.15)/100$. Absolute zero = 0 K = -273.15 °C.
- **Expansion relations:** $\alpha : \beta : \gamma = 1 : 2 : 3$. For an ideal gas at constant pressure, $\gamma = 1/T$.
- **Thermal stress:** if a rod is not allowed to expand, stress = $Y\alpha \Delta T$. That is why railway tracks have gaps.
- **Importance of anomalous expansion:** lakes freeze from the top; water at 4 °C (densest) stays at the bottom, so fish survive winter.
- **Heat capacity** $S = \Delta Q/\Delta T$; **specific heat** $c = \Delta Q/(m\Delta T)$; **molar specific heat** $C = \Delta Q/(n\Delta T)$. Water has high c (4200 J/kg K) so it is used as coolant and moderates climate.
- **Calorimetry:** heat lost by hot body = heat gained by cold body (no loss to surroundings).
- **Change of state:** temperature stays constant while melting/boiling. L_f (ice) = 3.33×10^5 J/kg, L_v (water) = 2.256×10^6 J/kg. Higher pressure raises boiling point (pressure cooker) and lowers melting point of ice (regelation).
- **Conduction:** $H = KA \Delta T/x$; thermal resistance $R = x/KA$. **Convection:** sea breeze (day), land breeze (night), trade winds. **Radiation:** fastest, needs no medium.
- **Black body** absorbs all radiation; good absorbers are good emitters. Stefan-Boltzmann: $P = e\sigma AT^4$, $\sigma = 5.67 \times 10^{-8}$ W m⁻² K⁻⁴. **Newton's law of cooling:** rate of cooling $\propto (T - T_s)$.

$$\Delta L = L\alpha\Delta T; \beta = 2\alpha; \gamma = 3\alpha$$

$$Q = mc\Delta T, Q = mL$$

$$\text{Conduction: } H = KA(T_1 - T_2)/x$$

$$\text{Wien: } \lambda_m T = b = 2.9 \times 10^{-3} \text{ m K}$$

$$\text{Stefan: } E = \sigma T^4$$

🔗 Quick 1-mark questions

At what temperature are °C and °F equal?

-40°

Why is water used in car radiators?

High specific heat

Which star is hotter: red or blue?

Blue (shorter λ_m)

📎 Solved problems (9)

Q1 How much heat is needed to warm 2 kg water from 20°C to 70°C? ($c = 4200 \text{ J/kg K}$)

$$1. Q = mc\Delta T = 2 \times 4200 \times 50$$

✓ Answer: $Q = 4.2 \times 10^5 \text{ J}$

Q2 A 1 m steel rod ($\alpha = 1.2 \times 10^{-5} / \text{K}$) is heated by 100 K. Find its expansion.

$$1. \Delta L = L\alpha\Delta T = 1 \times 1.2 \times 10^{-5} \times 100$$

✓ Answer: $\Delta L = 1.2 \times 10^{-3} \text{ m} = 1.2 \text{ mm}$

Q3 The Sun's radiation peaks at 500 nm. Estimate its surface temperature.

$$1. T = b/\lambda_m = 2.9 \times 10^{-3} / (5 \times 10^{-7})$$

✓ Answer: $T \approx 5800 \text{ K}$

Q4 Convert normal body temperature 98.6 °F into °C.

$$1. C = (F - 32) \times 5/9 = 66.6 \times 5/9$$

✓ Answer: 37°C

Q5 0.2 kg water at 80 °C is mixed with 0.3 kg water at 20 °C. Find the final temperature.

$$1. \text{Heat lost} = \text{heat gained: } 0.2 \times c \times (80 - T) = 0.3 \times c \times (T - 20)$$

$$2. 16 - 0.2T = 0.3T - 6 \Rightarrow 0.5T = 22$$

✓ Answer: $T = 44^\circ \text{C}$

Q6 How much heat changes 1 kg ice at 0 °C into water at 20 °C? ($L_f = 3.33 \times 10^5 \text{ J/kg}$)

$$1. \text{Melting: } mL = 3.33 \times 10^5 \text{ J}$$

$$2. \text{Heating: } mc\Delta T = 1 \times 4200 \times 20 = 0.84 \times 10^5 \text{ J}$$

✓ Answer: $4.17 \times 10^5 \text{ J}$

Q7 A glass window ($K = 0.8 \text{ W/m K}$) of area 2 m^2 and thickness 5 mm has 10°C difference across it. Find heat flow per second.

1. $H = KA\Delta T/x = 0.8 \times 2 \times 10/0.005$

✓ Answer: 3200 W

Q8 0.1 kg metal at 100°C is dropped into 0.2 kg water at 20°C . Final temperature is 25°C . Find c of metal.

1. $0.1 \times c \times 75 = 0.2 \times 4200 \times 5 = 4200$

2. $c = 4200/7.5$

✓ Answer: 560 J/kg K

Q9 If the absolute temperature of a black body doubles, how does its radiated power change?

1. $P \propto T^4 \Rightarrow (2)^4$

✓ Answer: 16 times



Long answer questions (5 marks)

L1 Explain the three modes of heat transfer with examples. Define thermal conductivity.

Answer:

1. **Conduction:** heat passes from particle to particle without the particles moving from their places; mainly in solids. Example: a metal spoon in hot tea becomes hot.
2. Rate of heat flow through a slab: $H = KA(T_1 - T_2)/x$, where K is the **thermal conductivity** ($\text{W m}^{-1} \text{K}^{-1}$). Metals have large K ; wood, air and wool have small K (insulators).
3. **Convection:** heat carried by actual movement of hot fluid (liquids and gases). Examples: sea and land breezes, boiling water circulation, trade winds.
4. **Radiation:** heat travels as electromagnetic waves, needs no medium and is fastest. Example: heat of the Sun reaching Earth.
5. Thermos flask stops all three: vacuum (conduction and convection), silvered walls (radiation).

★ Heat always flows from hot to cold by one or more of these modes.

L2 What is thermal expansion? Derive the relation between α , β and γ .

Answer:

1. Most substances expand on heating because atoms vibrate with larger amplitude and move apart.
2. Linear: $\Delta L = L\alpha\Delta T$; Area: $\Delta A = A\beta\Delta T$; Volume: $\Delta V = V\gamma\Delta T$.
3. Take a cube of side L . After heating: $L' = L(1 + \alpha\Delta T)$.
4. Area $A' = L'^2(1 + \alpha\Delta T)^2 \approx A(1 + 2\alpha\Delta T)$ (neglect α^2) $\Rightarrow \beta = 2\alpha$.
5. Volume $V' = L'^3(1 + \alpha\Delta T)^3 \approx V(1 + 3\alpha\Delta T) \Rightarrow \gamma = 3\alpha$. So $\alpha : \beta : \gamma = 1 : 2 : 3$.

★ Expansion must be allowed for in bridges, rails and overhead wires.

✓ Multiple choice questions (MCQs)

1. Water has maximum density at

- (a) 0 °C (b) 4 °C
(c) 100 °C (d) -4 °C

2. Relation between α , β and γ is

- (a) 1:2:3 (b) 3:2:1
(c) 1:1:1 (d) 1:3:2

3. Heat from the Sun reaches Earth by

- (a) conduction (b) convection
(c) radiation (d) all three

4. Temperature at which °C and °F readings are equal is

- (a) 0° (b) -40°
(c) 40° (d) 100°

5. During melting, temperature of ice

- (a) rises (b) falls
(c) stays constant (d) doubles

6. Sea breeze is an example of

- (a) conduction (b) convection
(c) radiation (d) evaporation

7. If absolute temperature doubles, radiated power becomes

- (a) 2 times (b) 4 times
(c) 8 times (d) 16 times

8. Wien's law: $\lambda_{\text{max}} \times T =$

- (a) constant (b) zero
(c) T^2 (d) σ

Answer key: 1-(b) Anomalous expansion. | 2-(a) $\beta = 2\alpha$, $\gamma = 3\alpha$. | 3-(c) No medium needed. | 4-(b) $C = F = -40$. | 5-(c) Heat used as latent heat. | 6-(b) Moving air carries heat. | 7-(d) $P \propto T^4$. | 8-(a) $b = 2.9 \times 10^{-3} \text{ m K}$.

11 Thermodynamics

Class XI · Unit VIII

In simple words

- **Zeroth law:** if A and B are each in thermal equilibrium with C, then A and B are in equilibrium with each other. This defines temperature.
- **First law:** heat given = increase in internal energy + work done by the system (energy conservation).
- **Isothermal:** temperature constant. **Adiabatic:** no heat exchange. **Cyclic:** system returns to start, so $\Delta U = 0$.
- **Reversible** process can be retraced exactly; real processes are **irreversible**.
- **Second law:** heat cannot fully change into work in a cycle (Kelvin–Planck); heat does not flow from cold to hot by itself (Clausius).

Remember in one line

- | | |
|--|---|
| 1 Zeroth law defines temperature. | 2 First law: heat given = internal energy rise + work done. |
| 3 Isothermal: T constant; adiabatic: no heat exchange. | 4 In a cycle, $\Delta U = 0$. |
| 5 Heat flows by itself only from hot to cold (second law). | 6 No engine can be 100% efficient. |

Theory in detail

Thermal equilibrium and temperature

When two bodies in contact stop exchanging heat, they are in **thermal equilibrium** — they have the same temperature. The zeroth law lets us use a thermometer: if the thermometer (C) is in equilibrium with body A and with body B separately, A and B are at the same temperature.

Internal energy

Internal energy (U) is the total of kinetic and potential energies of all molecules inside a system. For an ideal gas it depends only on temperature. It can be changed in two ways: by giving heat (Q) or by doing work (W). The first law connects these: $Q = \Delta U + W$.

Why the second law?

The first law says energy is conserved but does not tell the **direction** of a process. It would allow a cold cup of tea to become hotter by taking heat from the air — which never happens. The second law adds this rule: heat flows by itself only from hot to cold, and no engine can turn all heat into work.

Heat, work and internal energy

Heat and work are two ways of transferring energy. When you rub your hands, work increases their internal energy and they warm up. When you heat water, heat increases its internal energy.

Neither heat nor work is 'stored' in a body — what is stored is internal energy. The first law is simply energy conservation including heat.

Reversible, irreversible and cyclic processes

A **reversible** process can be reversed exactly, so both system and surroundings return to their original states; it must be very slow and free of friction. Real processes like free expansion, heat flow through a finite temperature difference, and friction are **irreversible**. In a **cyclic** process the system returns to its initial state, so $\Delta U = 0$ and net heat absorbed equals net work done.

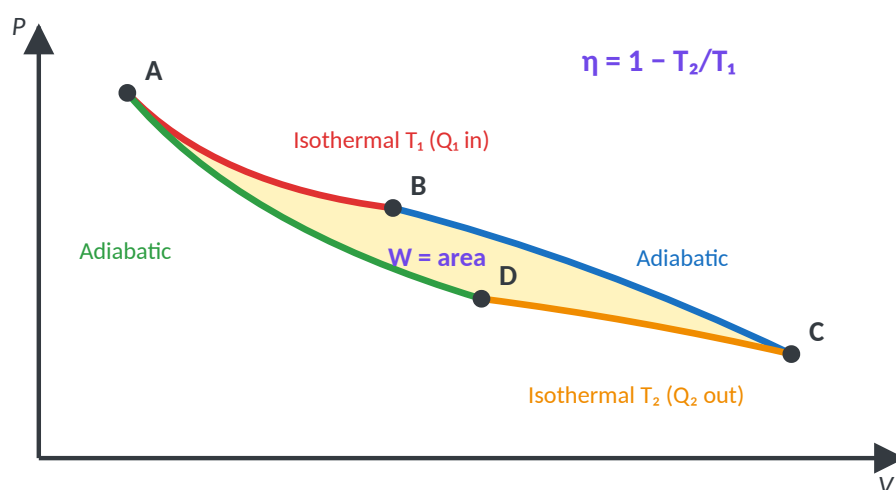


 Fig. 11.1: Carnot cycle on a P-V diagram: net work = enclosed area

More theory - explained

- **System & state variables:** P, V, T describe the state. Intensive (T, P, density) do not depend on size; extensive (V, mass, U) do.
- **State function vs path function:** internal energy U depends only on state; heat Q and work W depend on the path.
- **Sign convention:** Q positive when heat is absorbed; W positive when work is done *by* the system.
- **Work on P-V diagram** = area under the curve. In a cycle, net work = area enclosed.
- **First law in special processes:** Isothermal $\Delta U = 0 \Rightarrow Q = W$. Adiabatic $Q = 0 \Rightarrow W = -\Delta U$ (gas cools on expansion). Isochoric $W = 0 \Rightarrow Q = \Delta U$. Isobaric $Q = \Delta U + P\Delta V$.
- **Adiabatic relations:** $PV^\gamma = \text{const}$, $TV^{\gamma-1} = \text{const}$, $W = (P_1V_1 - P_2V_2)/(\gamma - 1)$. Mayer's relation: $C_p - C_v = R$.
- **Heat engine:** takes Q_1 from source, does work W, rejects Q_2 to sink. $\eta = W/Q_1 = 1 - Q_2/Q_1$.
- **Refrigerator:** works in reverse. Coefficient of performance $\beta = Q_2/W = T_2/(T_1 - T_2)$.
- **Carnot cycle:** isothermal expansion $\rightarrow$ adiabatic expansion $\rightarrow$ isothermal compression $\rightarrow$ adiabatic compression. **Carnot's theorem:** no engine between two temperatures is more efficient than a reversible Carnot engine.
- **Reversible process** must be quasi-static (very slow) with no friction or heat loss. All natural processes are irreversible.

Key formulas

$$Q = \Delta U + W$$

$$W = P\Delta V \text{ (constant pressure)}$$

$$\text{Isothermal: } W = nRT \ln(V_2/V_1)$$

$$\text{Adiabatic: } PV^\gamma = \text{constant}$$

$$\text{Carnot efficiency } \eta = 1 - T_2/T_1$$

Quick 1-mark questions

In which process is $\Delta U = 0$ for an ideal gas?

Isothermal (and cyclic)

Can any engine be 100% efficient?

No, needs $T_2 = 0 \text{ K}$

Why does air from a burst tyre feel cold?

Sudden adiabatic expansion

Solved problems (7)

Q1 500 J of heat is given to a gas and it does 200 J of work. Find the change in internal energy.

1. $Q = \Delta U + W$

2. $\Delta U = 500 - 200$

✓ Answer: $\Delta U = 300 \text{ J}$ (increases)

Q2 A Carnot engine works between 500 K and 300 K. Find its efficiency.

1. $\eta = 1 - T_2/T_1 = 1 - 300/500 = 0.4$

✓ Answer: $\eta = 40\%$

Q3 A gas expands from 2 L to 5 L at constant pressure 10^5 Pa . Find work done.

1. $W = P\Delta V = 10^5 \times 3 \times 10^{-3}$

✓ Answer: 300 J

Q4 1 mol ideal gas at 300 K doubles its volume isothermally. Find W , Q and ΔU .

1. $W = nRT \ln 2 = 8.314 \times 300 \times 0.693 \approx 1728 \text{ J}$

2. Isothermal $\Rightarrow \Delta U = 0 \Rightarrow Q = W$

✓ Answer: $W = Q \approx 1728 \text{ J}$, $\Delta U = 0$

Q5 An engine takes 1000 J and rejects 600 J per cycle. Find work and efficiency.

1. $W = 1000 - 600 = 400 \text{ J}$

2. $\eta = 400/1000$

✓ Answer: 400 J, 40%

Q6 A refrigerator keeps its inside at 270 K in a 300 K room. Find its ideal COP.

1. $\beta = T_2/(T_1 - T_2) = 270/30$

✓ Answer: 9

2. In an isothermal process, for an ideal gas
- (a) $\Delta U = 0$ (b) $Q = 0$
(c) $W = 0$ (d) $P = \text{constant}$

3. In an adiabatic process

- | | |
|----------------|----------------|
| (a) T constant | (b) $Q = 0$ |
| (c) V constant | (d) P constant |

4. Efficiency of a Carnot engine between 400 K and 300 K is

- | | |
|---------|----------|
| (a) 25% | (b) 50% |
| (c) 75% | (d) 100% |

5. Work done in a cyclic process equals

- | | |
|----------------|--------------------------------|
| (a) zero | (b) area enclosed on P-V graph |
| (c) ΔU | (d) PV |

6. Zeroth law introduces the concept of

- | | |
|-----------------|-------------|
| (a) heat | (b) work |
| (c) temperature | (d) entropy |

7. $C_P - C_V$ for an ideal gas equals

- | | |
|---------|--------|
| (a) R | (b) 2R |
| (c) R/2 | (d) 0 |

8. When a gas expands adiabatically, its temperature

- | | |
|----------------|------------------|
| (a) rises | (b) falls |
| (c) stays same | (d) becomes zero |

Answer key: 1-(c) $Q = \Delta U + W$. | 2-(a) U depends only on T. | 3-(b) No heat exchange. | 4-(a) $1 - 300/400$. | 5-(b) Net work = enclosed area. | 6-(c) Thermal equilibrium. | 7-(a) Mayer's relation. | 8-(b) $W = -\Delta U$.

12 Kinetic Theory of Gases

Class XI · Unit IX

In simple words

- An **ideal gas** obeys $PV = nRT$ at all temperatures and pressures.
- Gas molecules are tiny, move randomly, and collide elastically. Their collisions with walls create **pressure**.
- Average kinetic energy of a molecule depends only on temperature: $KE = (3/2)k_B T$.
- **Degrees of freedom**: monatomic = 3, diatomic = 5. **Equipartition**: each degree gets $\frac{1}{2}k_B T$ energy.
- **Mean free path**: average distance a molecule travels between two collisions.

Remember in one line

- 1 Gas molecules move randomly and collide elastically.
- 2 Pressure is caused by molecules hitting the walls.
- 3 Temperature measures average KE of molecules.
- 4 $v_{rms} = \sqrt{(3RT/M)}$: lighter gases move faster.
- 5 Each degree of freedom gets $\frac{1}{2}k_B T$ energy.
- 6 Mean free path is the average distance between collisions.

Theory in detail

Molecular nature of matter

All matter is made of tiny molecules in constant motion. In solids they vibrate about fixed points, in liquids they move around each other, and in gases they fly freely and far apart. Kinetic theory explains big-scale properties of gases (pressure, temperature) from the motion of these tiny molecules.

Law of equipartition of energy

In thermal equilibrium, the total energy of a molecule is shared equally among all its degrees of freedom, each getting $\frac{1}{2}k_B T$. So a monatomic molecule has $(3/2)k_B T$ and a diatomic molecule $(5/2)k_B T$ at room temperature. This law lets us calculate specific heats of gases directly.

Brownian motion

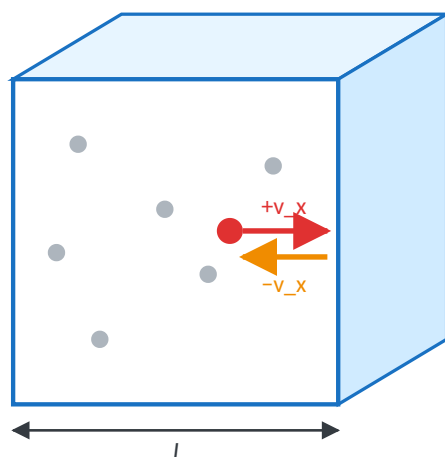
Pollen grains in water move in a zig-zag random way. This happens because water molecules hit them unequally from different sides. Brownian motion is strong evidence that molecules exist and are always moving; it increases with temperature and decreases with size of the particle and viscosity.

Work done in compressing a gas

When a gas is compressed by a piston, work is done on it: $W = \int P dV$ (area under P–V curve). At constant pressure $W = P\Delta V$. In isothermal compression the work goes out as heat; in adiabatic compression it raises the temperature (a bicycle pump becomes warm).

Mean free path

Gas molecules keep colliding with each other. The **mean free path** is the average distance a molecule travels between two collisions: $\lambda = 1/(\sqrt{2}\pi nd^2)$. It is larger when molecules are fewer (low pressure) or smaller. In air at normal pressure it is about 10^{-7} m. This is why a smell takes time to spread, even though molecules move at hundreds of m/s.



One hit: $\Delta p = 2mv_x$

Hits per second = $v_x / 2l$

Force = mv_x^2 / l

$P = \frac{1}{3} \rho v_{rms}^2$

KE per molecule = $(3/2) k_B T$

 Fig. 12.1: A gas molecule hitting the wall of a cube produces pressure

More theory - explained

- **Gas laws:** Boyle ($PV = \text{const}$ at const T), Charles ($V \propto T$ at const P), Gay-Lussac ($P \propto T$ at const V), Avogadro (equal volumes contain equal molecules). Together: $PV = nRT = Nk_B T$.
- **Real gases** behave like ideal gases at low pressure and high temperature.
- **Assumptions of KTG:** molecules are tiny points, move randomly, collide elastically, no forces except during collision, collision time negligible.
- **Pressure derivation idea:** each wall collision changes momentum by $2mv_x$; adding for all molecules gives $P = \frac{1}{3} nm \bar{v}^2 = \frac{1}{3} \rho v_{rms}^2$.
- **Meaning of temperature:** average KE per molecule = $(3/2)k_B T$; per mole = $(3/2)RT$. It does not depend on the kind of gas.
- **Three speeds:** $v_{rms} = \sqrt{(3RT/M)} > v_{avg} = \sqrt{(8RT/\pi M)} > v_{mp} = \sqrt{(2RT/M)}$.
- **Degrees of freedom:** monatomic 3 (translation); diatomic 5 (3 trans + 2 rot), 7 at high T (with vibration); non-linear polyatomic 6.
- **Specific heats:** $C_V = (f/2)R$, $C_P = (f/2 + 1)R$, $\gamma = 1 + 2/f$. Solids: $C = 3R$ (Dulong–Petit).
- **Mean free path** $\lambda = 1/(\sqrt{2} \pi nd^2)$: larger at low pressure (fewer molecules). $N_A = 6.02 \times 10^{23}$; 1 mole of gas at STP occupies 22.4 L.

Key formulas

$$PV = nRT, R = 8.314 \text{ J/mol K}$$

$$P = \frac{1}{3} \rho v_{rms}^2$$

$$v_{rms} = \sqrt{(3RT/M)}$$

Monatomic $\gamma = 5/3$; Diatomic $\gamma = 7/5$

$$\lambda = 1/(\sqrt{2} \text{ nm}^{-2})$$

⚡ Quick 1-mark questions

At what temperature is v_{rms} zero (classically)?

0 K

Does average KE depend on type of gas?

No, only on T

Degrees of freedom of a diatomic molecule at room temperature?

5

📎 Solved problems (8)

Q1 Find the rms speed of oxygen molecules at 300 K. ($M = 0.032 \text{ kg/mol}$)

$$1. v_{\text{rms}} = \sqrt{(3RT/M)} = \sqrt{(3 \times 8.314 \times 300 / 0.032)}$$

$$2. = \sqrt{(2.34 \times 10^5)}$$

✓ Answer: $v_{\text{rms}} \approx 484 \text{ m/s}$

Q2 Find γ for a monatomic gas using equipartition.

$$1. f = 3 \Rightarrow C_V = (f/2)R = 3R/2$$

$$2. C_P = C_V + R = 5R/2$$

$$3. \gamma = C_P/C_V$$

✓ Answer: $\gamma = 5/3 \approx 1.67$

Q3 Compare v_{rms} of H_2 and O_2 at the same temperature.

$$1. v_{\text{rms}} \propto 1/\sqrt{M} \Rightarrow v_{\text{H}}/v_{\text{O}} = \sqrt{(32/2)}$$

✓ Answer: 4 : 1

Q4 At what temperature will v_{rms} of a gas be double its value at 27°C ?

$$1. v \propto \sqrt{T} \Rightarrow T_2 = 4 \times 300 = 1200 \text{ K}$$

✓ Answer: 1200 K = 927°C

Q5 Find the average KE of a gas molecule at 300 K. ($k_B = 1.38 \times 10^{-23} \text{ J/K}$)

$$1. \text{KE} = (3/2)k_B T = 1.5 \times 1.38 \times 10^{-23} \times 300$$

✓ Answer: $6.21 \times 10^{-21} \text{ J}$

Q6 Find the number of molecules in 1 m^3 of gas at STP.

$$1. N = PV/k_B T = 1.013 \times 10^5 / (1.38 \times 10^{-23} \times 273)$$

✓ Answer: $\approx 2.69 \times 10^{25}$

Q7 Gas of density 1.3 kg/m^3 has $v_{\text{rms}} = 500 \text{ m/s}$. Find its pressure.

1. $P = \frac{1}{3}\rho v_{\text{rms}}^2 = \frac{1}{3} \times 1.3 \times 2.5 \times 10^5$

✓ Answer: $\approx 1.08 \times 10^5 \text{ Pa}$

Q8 Find C_V , C_P and γ for a diatomic gas at room temperature.

1. $f = 5 \Rightarrow C_V = 5R/2 \approx 20.8 \text{ J/mol K}$

2. $C_P = 7R/2 \approx 29.1 \text{ J/mol K}$

3. $\gamma = 7/5$

✓ Answer: 20.8, 29.1 J/mol K; $\gamma = 1.4$

Long answer questions (5 marks)

L1 Derive the expression for pressure exerted by an ideal gas using kinetic theory.

Answer:

1. Consider N molecules each of mass m in a cube of side l . A molecule has velocity components (v_x, v_y, v_z) .
2. On hitting the wall normal to x , momentum change $= -mv_x - mv_x = -2mv_x$. Momentum given to wall $= 2mv_x$.
3. Time between successive hits on the same wall $= 2l/v_x$. Force by one molecule $= 2mv_x / (2l/v_x) = mv_x^2 / l$.
4. Total force $= (m/l) \sum v_x^2$. Pressure $P = F/l^2 = (m/l^3) \sum v_x^2 = (mN/V) \langle v_x^2 \rangle$.
5. By symmetry $\langle v_x^2 \rangle = \frac{1}{3} \langle v^2 \rangle \Rightarrow P = \frac{1}{3} (mN/V) v_{\text{rms}}^2 = \frac{1}{3} \rho v_{\text{rms}}^2$.

★ Gas pressure is caused by molecules bombarding the walls.

L2 Give the kinetic interpretation of temperature and derive $v_{\text{rms}} = \sqrt{(3RT/M)}$.

Answer:

1. From kinetic theory, $PV = \frac{1}{3}Nm v_{\text{rms}}^2 = \frac{2}{3}N(\frac{1}{2}mv_{\text{rms}}^2)$.
2. From ideal gas equation, $PV = Nk_B T$.
3. Equating: $\frac{1}{2}mv_{\text{rms}}^2 = (3/2)k_B T$ — average KE of a molecule is proportional to absolute temperature. This is the kinetic meaning of temperature.
4. For one mole: $\frac{1}{2}Mv_{\text{rms}}^2 = (3/2)RT \Rightarrow v_{\text{rms}} = \sqrt{(3RT/M)} = \sqrt{(3P/\rho)}$.
5. At $T = 0 \text{ K}$ the KE (classically) becomes zero.

★ Temperature is simply a measure of the average kinetic energy of molecules.

✓ Multiple choice questions (MCQs)

1. Average KE of a gas molecule is proportional to

- (a) P
- (b) V
- (c) T
- (d) $\sqrt{T}$

2. v_{rms} of a gas is proportional to

- (a) T
- (b) $\sqrt{T}$
- (c) T^2
- (d) $1/T$

3. Degrees of freedom of a monatomic gas molecule are

- (a) 1
- (b) 3
- (c) 5
- (d) 7

4. γ for a diatomic gas at room temperature is

- (a) 1.67
- (b) 1.40
- (c) 1.33
- (d) 1.00

5. Pressure of an ideal gas is $P =$

- (a) $\frac{1}{3}\rho v_{rms}^2$
- (b) $\frac{1}{2}\rho v_{rms}^2$
- (c) ρv_{rms}^2
- (d) $3\rho v_{rms}^2$

6. Avogadro's number is

- (a) 6.02×10^{23}
- (b) 1.38×10^{-23}
- (c) 8.314
- (d) 6.63×10^{-34}

7. Mean free path increases when pressure

- (a) increases
- (b) decreases
- (c) stays same
- (d) doubles

8. At same T, which gas has highest v_{rms} ?

- (a) O_2
- (b) N_2
- (c) H_2
- (d) CO_2

Answer key: 1-(c) $KE = (3/2)k_B T$. | 2-(b) $v = \sqrt{(3RT/M)}$. | 3-(b) Only translation. | 4-(b) 7/5. | 5-(a) Kinetic theory result. | 6-(a) Particles in 1 mole. | 7-(b) Fewer molecules. | 8-(c) Smallest M.

13 Oscillations

Class XI • Unit X

In simple words

- **Periodic motion** repeats after equal time intervals (time period T). Frequency $f = 1/T$.
- **SHM**: restoring force is proportional to displacement and opposite to it: $F = -kx$.
- Displacement in SHM: $x = A \sin(\omega t + \phi)$. ϕ is the **phase** (initial position).
- In SHM, energy keeps changing between KE and PE, but **total energy stays constant** $= \frac{1}{2}kA^2$.
- Simple pendulum time period depends on length and g , **not on mass**. A seconds pendulum has $T = 2 \text{ s}$ ($l \approx 1 \text{ m}$).

Remember in one line

- | | |
|--|---|
| 1 Periodic motion repeats after equal time intervals. | 2 In SHM, restoring force $\propto$ displacement ($F = -kx$). |
| 3 Speed is maximum at the mean position; acceleration at the extremes. | 4 Total energy in SHM stays constant. |
| 5 Pendulum period depends on length and g , not mass. | 6 Spring period $T = 2\pi\sqrt{m/k}$. |

Theory in detail

Restoring force

An oscillating body always has a **mean (equilibrium) position**. When displaced, a force acts to bring it back — the **restoring force**. Because of inertia, the body overshoots the mean position and goes to the other side, and the process repeats. If restoring force $\propto$ displacement, the motion is SHM.

Loaded spring

A mass m attached to a spring of force constant k , when pulled by x , feels $F = -kx$ (Hooke's law). Comparing with $F = -m\omega^2x$ gives $\omega = \sqrt{k/m}$ and $T = 2\pi\sqrt{m/k}$. The force constant k is the force needed per unit extension (N/m); a stiff spring has large k and oscillates faster.

Phase

Phase tells the state of motion (position and direction) of an oscillating particle at any instant. In $x = A \sin(\omega t + \phi)$, the quantity $(\omega t + \phi)$ is the phase and ϕ is the **initial phase (epoch)**. Two particles are 'in phase' if they move together, and 'out of phase' (π difference) if they move opposite.

Displacement as a function of time

In SHM, $x(t) = A \cos(\omega t + \phi)$ or $A \sin(\omega t + \phi)$. The graph is a smooth wave. Velocity and acceleration graphs are also sine curves, shifted by a quarter and half period. Because these functions repeat, SHM is the simplest periodic motion, and any complex vibration can be built from many SHMs.

Energy-displacement graph

If we plot energy against displacement in SHM, PE is a parabola ($\frac{1}{2}kx^2$) that is zero at the centre and maximum at the ends, KE is an inverted parabola, and total energy is a horizontal line. Energy keeps shifting between KE and PE twice every cycle, so KE and PE each have a frequency twice that of the oscillation.

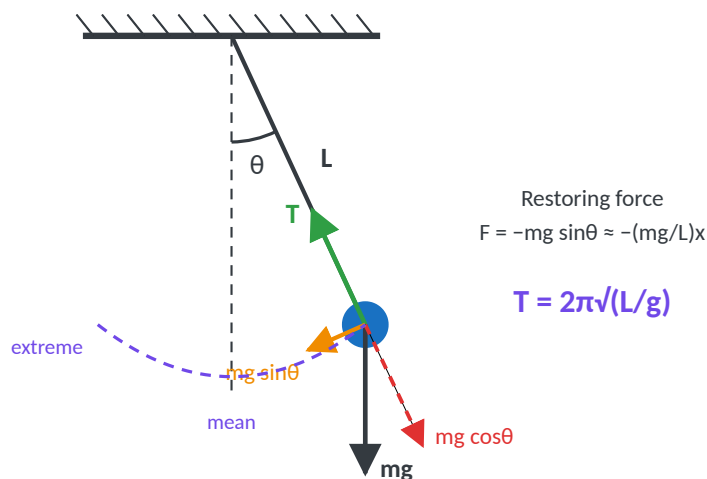


Fig. 13.1: Forces on a simple pendulum — $mg \sin \theta$ pulls it back to the mean position

More theory - explained

- **Periodic vs oscillatory:** every oscillation is periodic, but not every periodic motion is oscillatory (Earth's rotation is periodic, not to-and-fro).
- **Periodic functions:** $\sin \omega t$ and $\cos \omega t$ repeat after $2\pi/\omega$. Any periodic function can be written as a sum of sines and cosines.
- **SHM and circles:** SHM is the projection of uniform circular motion on a diameter.
- **Phase relations:** velocity is ahead of displacement by $\pi/2$; acceleration is ahead by π (opposite to x).
- **Energy:** $KE = \frac{1}{2}k(A^2 - x^2)$, $PE = \frac{1}{2}kx^2$, total = $\frac{1}{2}kA^2$. $KE = PE$ at $x = A/\sqrt{2}$. KE is max at mean position, PE max at extremes.
- **Springs:** series $1/k = 1/k_1 + 1/k_2$; parallel $k = k_1 + k_2$. Cutting a spring in half doubles k .
- **Pendulum derivation:** restoring force = $-mg \sin \theta \approx -mg\theta = -(mg/L)x$ (small θ) $\Rightarrow$ SHM with $\omega = \sqrt{g/L}$, $T = 2\pi\sqrt{L/g}$.
- **Pendulum facts:** T does not depend on mass or small amplitude. In a lift going up, $g_{\text{eff}} = g + a$ (T decreases). On Moon T is $\sqrt{6}$ times larger. At Earth's centre T is infinite.
- **Damped and forced oscillations:** amplitude slowly dies due to friction (damping). If a periodic force has the natural frequency, amplitude becomes very large — **resonance**.

Key formulas

$$x = A \sin(\omega t + \phi), \quad \omega = 2\pi/T$$

$$v = \omega\sqrt{A^2 - x^2}, \quad a = -\omega^2 x$$

$$\text{Spring: } T = 2\pi\sqrt{m/k}$$

Pendulum: $T = 2\pi\sqrt{l/g}$

$E = \frac{1}{2}m\omega^2A^2$

⚡ Quick 1-mark questions

Is Earth's motion round the Sun SHM?	Periodic, but not SHM
Phase difference between x and a in SHM?	π (180°)
Where is KE maximum in SHM?	At the mean position

📎 Solved problems (9)

Q1 A 0.5 kg mass hangs on a spring of $k = 50$ N/m. Find its time period.

1. $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.5/50} = 2\pi \times 0.1$

✓ Answer: $T \approx 0.628$ s

Q2 For $x = 0.05 \sin(10t)$ m, find amplitude, maximum speed and maximum acceleration.

1. $A = 0.05$ m, $\omega = 10$ rad/s

2. $v_{\max} = A\omega = 0.05 \times 10 = 0.5$ m/s

3. $a_{\max} = A\omega^2 = 0.05 \times 100 = 5$ m/s²

✓ Answer: $A = 5$ cm, $v_{\max} = 0.5$ m/s, $a_{\max} = 5$ m/s²

Q3 Find the length of a seconds pendulum. ($g = 9.8$ m/s²)

1. $T = 2$ s $\Rightarrow l = gT^2/4\pi^2 = 9.8 \times 4 / 39.48$

✓ Answer: $l \approx 0.993$ m ≈ 1 m

Q4 $x = 4 \cos(\pi t + \pi/4)$ cm. Find A , ω , T , f and initial phase.

1. Compare with $x = A \cos(\omega t + \phi)$

2. $A = 4$ cm, $\omega = \pi$ rad/s $\Rightarrow T = 2\pi/\omega = 2$ s, $f = 0.5$ Hz, $\phi = \pi/4$

✓ Answer: $A = 4$ cm, $T = 2$ s, $f = 0.5$ Hz, $\phi = \pi/4$

Q5 In SHM with $A = 5$ cm and $\omega = 2$ rad/s, find speed at $x = 3$ cm.

1. $v = \omega\sqrt{A^2 - x^2} = 2\sqrt{25 - 9} = 2 \times 4$

✓ Answer: 8 cm/s

Q6 A 0.2 kg mass on a spring ($k = 20$ N/m) oscillates with $A = 0.1$ m. Find total energy and maximum speed.

1. $E = \frac{1}{2}kA^2 = \frac{1}{2} \times 20 \times 0.01 = 0.1$ J

2. $v_{\max} = \sqrt{2E/m} = \sqrt{(0.2/0.2)}$

✓ Answer: 0.1 J; 1 m/s

Q7 Two springs of 100 N/m each are joined (a) in series (b) in parallel. Find k and period for a 2 kg mass in parallel.

1. Series: $k = 50 \text{ N/m}$; Parallel: $k = 200 \text{ N/m}$
2. $T = 2\pi\sqrt{(2/200)} = 2\pi \times 0.1$

✓ Answer: 50 N/m, 200 N/m; $T \approx 0.63 \text{ s}$

Q8 A seconds pendulum is taken to the Moon ($g/6$). Find its period there.

1. $T \propto 1/\sqrt{g} \Rightarrow T' = 2 \times \sqrt{6}$

✓ Answer: $\approx 4.9 \text{ s}$

Q9 If the length of a pendulum is made 4 times, what happens to its period?

1. $T \propto \sqrt{L} \Rightarrow \sqrt{4} = 2$

✓ Answer: It doubles

Long answer questions (5 marks)

L1 Define SHM. Derive expressions for velocity, acceleration and energy of a particle in SHM.

Answer:

1. **SHM:** periodic motion in which restoring force (and acceleration) is proportional to displacement from the mean position and directed towards it: $F = -kx$.
2. Displacement $x = A \sin(\omega t + \phi)$.
3. Velocity $v = dx/dt = A\omega \cos(\omega t + \phi) = \omega\sqrt{(A^2 - x^2)}$. Max ($A\omega$) at mean position, zero at extremes.
4. Acceleration $a = dv/dt = -A\omega^2 \sin(\omega t + \phi) = -\omega^2 x$. Max ($A\omega^2$) at extremes, zero at mean.
5. $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$; $PE = \int kx \, dx = \frac{1}{2}m\omega^2 x^2$.
6. Total $E = KE + PE = \frac{1}{2}m\omega^2 A^2$ — constant, independent of x .

★ Energy in SHM keeps changing form but the total stays constant.

L2 Derive the expression for the time period of a simple pendulum.

Answer:

1. A simple pendulum is a point mass m hung by a light, inextensible string of length L from a rigid support.
2. When displaced by small angle θ , forces are tension T and weight mg . Component $mg \cos\theta$ balances T ; component $mg \sin\theta$ acts as restoring force towards the mean position.
3. $F = -mg \sin\theta$. For small θ , $\sin\theta \approx \theta = x/L \Rightarrow F = -(mg/L)x$.
4. Since $F \propto -x$, the motion is SHM with $k = mg/L \Rightarrow \omega = \sqrt{(k/m)} = \sqrt{(g/L)}$.
5. $T = 2\pi\sqrt{(L/g)}$. T is independent of mass and (small) amplitude, depends on L and g .

★ A seconds pendulum ($T = 2 \text{ s}$) has $L \approx 1 \text{ m}$ on Earth.

✓ Multiple choice questions (MCQs)

1. In SHM, acceleration is proportional to
(a) velocity (b) displacement
(c) time (d) mass

3. Velocity in SHM is maximum at
(a) extreme position (b) mean position
(c) halfway (d) nowhere

5. If length of a pendulum is made 4 times, its period becomes
(a) 2 times (b) 4 times
(c) half (d) same

7. Total energy of SHM is proportional to
(a) A (b) A^2
(c) $1/A$ (d) $\sqrt{A}$

2. Time period of a simple pendulum is independent of
(a) length (b) g
(c) mass of bob (d) place

4. Length of a seconds pendulum is about
(a) 0.5 m (b) 1 m
(c) 2 m (d) 9.8 m

6. Phase difference between displacement and acceleration in SHM is
(a) 0 (b) $\pi/2$
(c) π (d) 2π

8. Time period of a mass-spring system is
(a) $2\pi\sqrt{k/m}$ (b) $2\pi\sqrt{m/k}$
(c) $2\pi\sqrt{mk}$ (d) $2\pi m/k$

Answer key: 1-(b) $a = -\omega^2 x$. | 2-(c) $T = 2\pi\sqrt{L/g}$. | 3-(b) All energy is KE there. | 4-(b) $T = 2$ s. | 5-(a) $T \propto \sqrt{L}$.
| 6-(c) $a = -\omega^2 x$. | 7-(b) $E = \frac{1}{2}kA^2$. | 8-(b) $\omega = \sqrt{k/m}$.

In simple words

- A **wave** carries energy from one place to another without carrying matter.
- **Transverse**: particles move perpendicular to wave (string, light). **Longitudinal**: particles move along the wave (sound).
- **Superposition**: when two waves meet, the net displacement is the sum of individual displacements.
- **Standing waves** form when two equal waves travel opposite ways. Points of zero motion are **nodes**, maximum motion are **antinodes**.
- Open pipe gives all harmonics; closed pipe gives only odd harmonics.
- **Beats**: periodic rise and fall of loudness when two nearly equal frequencies are sounded together.

Remember in one line

- | | |
|--|--|
| 1 Waves carry energy, not matter. | 2 Transverse: particles move across; longitudinal: along the wave. |
| 3 $v = f\lambda$ for all waves. | 4 Stationary waves have fixed nodes and antinodes. |
| 5 Closed pipes give only odd harmonics; open pipes give all. | 6 Beats = difference of two close frequencies. |

Theory in detail

How a wave travels

When one particle of a medium is disturbed, it pulls on its neighbours, which pull on theirs, and so the disturbance passes along. The particles themselves only vibrate about their mean positions; it is the **energy and pattern** that travel. A Mexican wave in a stadium is a good example — people stand and sit but do not run around the stadium.

Displacement relation

For a wave moving in +x direction: $y(x, t) = A \sin(kx - \omega t + \phi)$. Here A = amplitude, $k = 2\pi/\lambda$ (angular wave number), $\omega = 2\pi f$ (angular frequency). For a wave moving in -x direction the sign changes: $y = A \sin(kx + \omega t)$. Speed $v = \omega/k = f\lambda$.

Fundamental and harmonics

The lowest frequency a string or pipe can vibrate with is the **fundamental** or first harmonic. Higher frequencies that are whole-number multiples are **harmonics** (overtones). The mixture of harmonics gives each instrument its unique sound quality (timbre), so a violin and a flute playing the same note sound different.

Principle of superposition

When two or more waves pass through the same point, the resultant displacement is the algebraic sum of individual displacements: $y = y_1 + y_2 + \dots$. After crossing, each wave continues unchanged. Superposition explains interference, standing waves and beats.

Beats explained

When two sounds of slightly different frequencies f_1 and f_2 are heard together, the waves are sometimes in phase (loud) and sometimes out of phase (soft). This regular rise and fall in loudness is called **beats**. The number of beats per second = $|f_1 - f_2|$. The human ear can distinguish beats only if this difference is less than about 10 Hz.

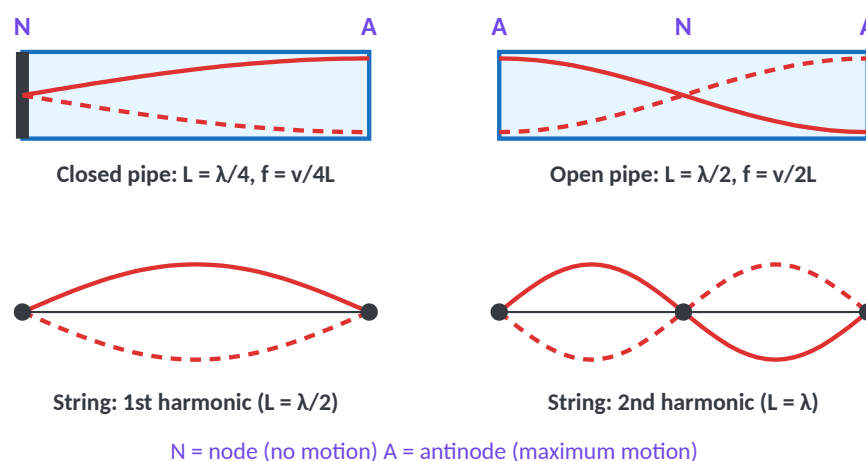


 Fig. 14.1: Standing waves in pipes and strings

More theory - explained

- **Mechanical waves** need a medium (sound, water waves); **EM waves** do not. Terms: amplitude A , wavelength λ , frequency f , wave number $k = 2\pi/\lambda$, $\omega = 2\pi f$.
- **Progressive wave** $y = A \sin(kx - \omega t + \phi)$, speed $v = \omega/k$. Phase difference = $(2\pi/\lambda) \times \text{path difference}$.
- **Speed of sound**: Newton assumed isothermal, $v = \sqrt{P/\rho} \approx 280 \text{ m/s}$ (wrong). **Laplace correction** (adiabatic): $v = \sqrt{\gamma P/\rho} \approx 331 \text{ m/s}$. $v \propto \sqrt{T}$; pressure has no effect; humidity increases v .
- **Speeds**: string $\sqrt{T/\mu}$; longitudinal in gas $\sqrt{B/\rho}$; in a solid rod $\sqrt{Y/\rho}$.
- **Reflection**: at a rigid end the wave reflects with phase change π (crest $\rightarrow$ trough); at an open end no phase change.
- **Standing wave** $y = 2A \sin kx \cos \omega t$. Node to node = $\lambda/2$, node to antinode = $\lambda/4$. Energy does not travel.
- **Laws of a stretched string** (sonometer): $f \propto 1/L$ (fixed T), $f \propto \sqrt{T}$ (fixed L), $f \propto 1/\sqrt{\mu}$.
- **Pipes**: closed pipe $L = (2n - 1)\lambda/4 \rightarrow$ only odd harmonics; open pipe $L = n\lambda/2 \rightarrow$ all harmonics. End correction $e \approx 0.3d$; **resonance tube**: $v = 2f(l_2 - l_1)$.
- **Beats uses**: tuning musical instruments, finding unknown frequency. Loading a prong with wax lowers f ; filing raises f .

Key formulas

$$v = f\lambda$$

$$y = A \sin(kx - \omega t), k = 2\pi/\lambda$$

$$\text{String: } v = \sqrt{T/\mu}, f_n = nv/2L$$

$$\text{Open pipe: } f_n = nv/2L; \text{ Closed pipe: } f = (2n-1)v/4L$$

$$\text{Beat frequency} = |f_1 - f_2|$$

Quick 1-mark questions

Can sound travel in vacuum?

No, it needs a medium

Distance between two consecutive nodes?

$\lambda/2$

Which harmonics are present in a closed pipe?

Only odd harmonics

Solved problems (9)

Q1 A 1 m string with tension 100 N and mass per length 0.01 kg/m. Find the wave speed and fundamental frequency.

1. $v = \sqrt{T/\mu} = \sqrt{(100/0.01)} = 100 \text{ m/s}$

2. $f_1 = v/2L = 100/2$

✓ Answer: $v = 100 \text{ m/s}$, $f_1 = 50 \text{ Hz}$

Q2 Find the fundamental frequency of a 0.5 m pipe ($v = 340 \text{ m/s}$) when (a) closed at one end, (b) open at both ends.

1. (a) $f = v/4L = 340/2 = 170 \text{ Hz}$

2. (b) $f = v/2L = 340/1 = 340 \text{ Hz}$

✓ Answer: Closed: 170 Hz, Open: 340 Hz

Q3 Two tuning forks of 256 Hz and 260 Hz are sounded together. How many beats per second?

1. Beats = $260 - 256$

✓ Answer: 4 beats per second

Q4 $y = 0.05 \sin(20\pi t - \pi x) \text{ m}$. Find A, f, λ and v.

1. $A = 0.05 \text{ m}$; $\omega = 20\pi \Rightarrow f = 10 \text{ Hz}$

2. $k = \pi \Rightarrow \lambda = 2\pi/k = 2 \text{ m}$

3. $v = f\lambda = 10 \times 2$

✓ Answer: $A = 5 \text{ cm}$, $f = 10 \text{ Hz}$, $\lambda = 2 \text{ m}$, $v = 20 \text{ m/s}$

Q5 Speed of sound is 332 m/s at 0 °C. Find it at 27 °C.

1. $v \propto \sqrt{T} \Rightarrow v = 332\sqrt{(300/273)} = 332 \times 1.048$

✓ Answer: $\approx 348 \text{ m/s}$

Q6 In a resonance tube with a 512 Hz fork, $l_1 = 16$ cm, $l_2 = 50$ cm. Find v and end correction.

1. $v = 2f(l_2 - l_1) = 2 \times 512 \times 0.34 \approx 348$ m/s

2. $e = (l_2 - 3l_1)/2 = (50 - 48)/2$

✓ Answer: $v \approx 348$ m/s, $e = 1$ cm

Q7 A sonometer wire resonates with 256 Hz at 60 cm. Find the length for 384 Hz.

1. $f \propto 1/L \Rightarrow L = 60 \times 256/384$

✓ Answer: 40 cm

Q8 Fork A (256 Hz) gives 4 beats/s with fork B. On waxing B, beats become 6/s. Find f_B .

1. $f_B = 252$ or 260 Hz

2. Wax lowers f_B . Beats increase $\Rightarrow f_B$ moved away from 256 $\Rightarrow$ it was below

✓ Answer: 252 Hz

Q9 Two points are 0.5 m apart on a wave of $\lambda = 2$ m. Find the phase difference.

1. $\Delta\phi = (2\pi/\lambda) \times \Delta x = (2\pi/2) \times 0.5$

✓ Answer: $\pi/2$



Long answer questions (5 marks)

L1 Discuss the formation of stationary waves in a string fixed at both ends. Show that all harmonics are present.

Answer:

1. A wave $y_1 = A \sin(kx - \omega t)$ reflects from a fixed end as $y_2 = A \sin(kx + \omega t)$.

2. By superposition: $y = y_1 + y_2 = 2A \sin kx \cos \omega t$. Amplitude $2A \sin kx$ depends on position — this is a stationary wave.

3. **Nodes** (amplitude zero): $\sin kx = 0 \Rightarrow x = 0, \lambda/2, \lambda, \dots$ **Antinodes** (max amplitude): $x = \lambda/4, 3\lambda/4, \dots$

4. Both ends of the string are nodes $\Rightarrow L = n\lambda/2 \Rightarrow \lambda_n = 2L/n$.

5. $f_n = v/\lambda_n = nv/2L = (n/2L)\sqrt{T/\mu}$, $n = 1, 2, 3 \dots$ So $f_1 : f_2 : f_3 = 1 : 2 : 3$ — all harmonics.



A stretched string produces a rich sound because it has all harmonics.

L2 Explain the modes of vibration of air column in (a) closed pipe and (b) open pipe.

Answer:

1. In pipes, a closed end is always a **node** and an open end is an **antinode**.

2. **(a) Closed pipe** (one end closed): node at closed end, antinode at open end. Fundamental: $L = \lambda/4 \Rightarrow f_1 = v/4L$.

3. Next modes: $L = 3\lambda/4, 5\lambda/4 \dots \Rightarrow f = 3v/4L, 5v/4L \dots$ Only **odd harmonics** (1 : 3 : 5).
4. **(b) Open pipe:** antinodes at both ends. Fundamental: $L = \lambda/2 \Rightarrow f_1 = v/2L$.
5. Next modes: $L = \lambda, 3\lambda/2 \dots \Rightarrow f = 2v/2L, 3v/2L \dots$ **All harmonics** (1 : 2 : 3).
6. For the same length, fundamental of open pipe is twice that of closed pipe; open pipe sounds richer.

★ That is why flutes (open pipes) sound richer than closed pipes.

✓ Multiple choice questions (MCQs)

1. Sound waves in air are

- (a) transverse (b) longitudinal
(c) electromagnetic (d) stationary only

2. Distance between two consecutive nodes is

- (a) λ (b) $\lambda/2$
(c) $\lambda/4$ (d) 2λ

3. A closed pipe produces

- (a) all harmonics (b) only odd harmonics
(c) only even harmonics (d) no harmonics

4. Two forks of 256 Hz and 262 Hz give beats per second =

- (a) 2 (b) 4
(c) 6 (d) 518

5. Speed of a wave on a string is

- (a) $\sqrt{T/\mu}$ (b) $\sqrt{(\mu/T)}$
(c) $T\mu$ (d) T/μ

6. Laplace correction assumes sound propagation is

- (a) isothermal (b) adiabatic
(c) isobaric (d) isochoric

7. Speed of sound in air increases with

- (a) pressure (b) temperature
(c) density only (d) amplitude

8. Wave $y = A \sin(kx - \omega t)$ moves

- (a) in +x direction (b) in -x direction
(c) upward (d) does not move

Answer key: 1-(b) Compressions and rarefactions. | 2-(b) Standing wave pattern. | 3-(b) Node at closed end.
| 4-(c) 262 - 256. | 5-(a) Standard result. | 6-(b) $v = \sqrt{(\gamma P/\rho)}$. | 7-(b) $v \propto \sqrt{T}$. | 8-(a) Sign of ωt and kx opposite.

Useful Constants & Exam Tips

Quick revision

Speed of light c	$3 \times 10^8 \text{ m/s}$
Gravitational constant G	$6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$
g on Earth	9.8 m/s^2
Radius of Earth R	$6.4 \times 10^6 \text{ m}$
Electron charge e	$1.6 \times 10^{-19} \text{ C}$
Electron mass m_e	$9.1 \times 10^{-31} \text{ kg}$
Planck's constant h	$6.63 \times 10^{-34} \text{ J s}$
ϵ_0	$8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2$
μ_0	$4\pi \times 10^{-7} \text{ T m/A}$
Gas constant R	8.314 J/mol K
Boltzmann k_B	$1.38 \times 10^{-23} \text{ J/K}$
Avogadro N_A	$6.02 \times 10^{23} / \text{mol}$
1 u	$1.66 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}$
1 eV	$1.6 \times 10^{-19} \text{ J}$

Exam tips

- Write the **given data** first, with units converted to SI ($\text{cm} \rightarrow \text{m}$, $\mu\text{C} \rightarrow 10^{-6} \text{ C}$).
- Write the **formula**, then substitute, then the answer **with unit**. Each step earns marks.
- Follow the **sign convention** carefully in mirror and lens sums.
- Draw a **neat labelled diagram** wherever possible (ray diagrams, circuits, graphs).
- Check your answer with **common sense**: drift velocity is tiny, escape speed $\sim 11 \text{ km/s}$, etc.
- Revise the yellow formula boxes daily – 10 minutes a day is enough.
- For derivations, remember the **starting idea** and the **final result**; the middle steps follow.

Based on the CHSE Odisha Physics syllabus 2023–24 • Reference: NCERT Physics Class XI & XII